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DS DYN FLUIDE 2023 - 2024.

Questions de cours :

Q1) $\operatorname{div} \overline{\overline{\sigma}} = -\vec{\nabla} p + \mu \Delta \vec{v} + (\lambda+\mu) \vec{\nabla} \operatorname{div} \vec{v}$

$\overline{\overline{\sigma}} = -p \overline{\overline{I}} + 2\mu \overline{\overline{D}} + \lambda \operatorname{div}(\vec{v}) \overline{\overline{I}}$

- $\frac{\partial}{\partial x_j}(-p\,\delta_{ij}) = -\frac{\partial p}{\partial x_i} = -(\vec{\nabla} p)_i$ (1)

- $\frac{\partial}{\partial x_j}\left(\frac{2\mu}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right) = \mu\left(\frac{\partial^2 v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i \partial x_j}\right)$

$= \mu(\Delta \vec{v})_i + \mu\, \vec{\nabla} \operatorname{div}(\vec{v})_i$ (1)

- $\lambda \frac{\partial}{\partial x_j}\left(\frac{\partial v_k}{\partial x_k}\right)\delta_{ij} = \lambda \frac{\partial}{\partial x_i}\frac{\partial v_k}{\partial x_k} = \lambda \frac{\partial}{\partial x_i}\frac{\partial v_j}{\partial x_j}$

$= \lambda(\vec{\nabla} \operatorname{div} \vec{v})_i$ (1)

Q2) Signification physique (2 points)

Q3) DAR : $\varepsilon \ll 1$ devant dérivée d'ordre 2 ordre. (1)

Q4) (i) $\rho \frac{d\vec{v}}{dt} = \operatorname{div} \overline{\overline{\sigma}} + \vec{f_v}$

(i)$\cdot \vec{v} \Rightarrow \rho \frac{de_c}{dt} = (\operatorname{div} \overline{\overline{\sigma}})\cdot \vec{v} + \vec{f_v} \cdot \vec{v}$

(1)

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0,5

FC $\quad \dfrac{\partial \rho\varphi}{\partial t} + \operatorname{div} \rho\varphi\vec{v} = \operatorname{div}\vec{\Phi_s} + S_v$

FNC $\quad \rho \dfrac{d\varphi}{dt} = \rho\left(\dfrac{\partial\varphi}{\partial t} + \vec{v}\cdot\vec{\nabla}\varphi\right) = \operatorname{div}\vec{\Phi_s} + S_v$

$$\frac{\partial \rho\varphi}{\partial t} + \operatorname{div} \rho\varphi\vec{v} = \left[\varphi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\varphi}{\partial t}\right] + \left[\rho\vec{v}\cdot\vec{\nabla}\varphi + \varphi \operatorname{div}\rho\vec{v}\right]$$

0,5 + 1

$$= \varphi\underbrace{\left(\frac{\partial\rho}{\partial t} + \operatorname{div}\rho\vec{v}\right)}_{=0 \text{ CM}} + \rho\left(\frac{\partial\varphi}{\partial t} + \vec{v}\cdot\vec{\nabla}\varphi\right)$$

1 FC : Pour numérique car ne nécessite pas CM dc ne se propage pas aux autres éq.

FNC : Pour résolution analytique car + simple.

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Q6) $U = \lambda \times f$ 0,5

variat° spatiale : λ 0,5 1,5

néglijeable .

Q7) CM : $\delta\rho f \frac{\partial \bar{\rho}}{\partial t} + \frac{\rho_0 U}{\lambda} \mathrm{div}\, \bar{v} + \cancel{\frac{\delta\rho U}{\lambda} \vec{v} \cdot \vec{\nabla}(\bar{\rho})} = 0.$

BQM : $\rho_0 \left[U f \frac{\partial \vec{v}}{\partial t} + \frac{U^2}{\lambda} \vec{v} \cdot \vec{\nabla}(\vec{v}) \right]$

$$= -\frac{\delta P}{\lambda} \vec{\nabla} \bar{P} + \frac{\mu U}{\lambda^2} \bar{\Delta} \vec{v} + \frac{(\lambda + \mu)}{\lambda^2} \vec{\nabla}\, \mathrm{div}\, \vec{v} \quad 1,5$$

CM : $\delta \rho f \sim \frac{\rho_0 U}{\lambda} \Longrightarrow \delta\rho = \frac{\rho_0 U}{\lambda f} = \frac{\rho_0 U}{c_0}$ 0,5 M

BQM : $\rho_0 U f \sim \frac{\delta P}{\lambda} \Rightarrow \delta P = \rho_0 U f \lambda = \rho_0 U c_0$ 0,5

Rem : $\frac{\delta P}{\delta \rho} = c_0^2$

Q8) $\Pi_1 = \frac{\mu \cancel{U}}{\lambda^2 \rho_0 \cancel{U} f} = \frac{\mu}{\lambda^2 \rho_0 f} = \frac{\mu f}{c_0^2 \rho_0}$

$\lambda = \frac{c_0}{f} = \frac{1500}{10^6} \sim 1,5 \times 10^{-3}$

$\Pi_1 \sim \frac{10^{-3}}{(1,5)^2 \times (10^{-3})^2 \times 10^3 \times 10^6}$

$\Pi_1 = \frac{10^{-3} \times 10^6}{(1,5)^2 \times (10^3)^2 \times 10^3}$

$\sim 10^{-6} \ll 1$

On peut négliger les termes visqueux . 1

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Q9) $\Pi_2 = \dfrac{\text{diff. therm}}{\text{insta}}$

$$= \frac{k\,\delta T}{d^2 \rho C_p \delta T f} = \frac{k}{d^2 \rho C_p f}.$$ 0,5

$\Pi_3 = \dfrac{\text{éch. visq.}}{\text{insta}}$ 0,5

$$= \frac{\mu U^2}{d^2 \rho C_p \delta T f}$$

$$\Pi_2 = \frac{k}{C_p \mu} \Pi_1 = \frac{k}{\nu} \Pi_1 = \frac{1}{P_r} \Pi_1$$ 0,5

$P_r = O(1) \Rightarrow \{\Pi_1 << 1 \Rightarrow \Pi_2 << 1\}$.

$\Pi_3 = \epsilon \Pi_1$ 0,5

Au final $\Pi_1 << 1 \Rightarrow \{\Pi_2, \Pi_3 << 1\}$ et donc

$$\rho C_p \left[\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla}(T)\right] = -\alpha T \left[\frac{\partial p}{\partial t} + \vec{v} \cdot \vec{\nabla}(p)\right]$$

Q10) Devient $\rho T \dfrac{ds}{dt} = 0$

et donc si so homogène initialement, entropie constante. 1

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Q11) Equations d'Euler (0,5)

Q12) • $\frac{\delta P}{\delta \rho} = c_0^2$ puisque s cste

• Idem Q7.

• $\partial \tilde{p} = c_0^2 \, \partial \tilde{\rho}$ et donc $\frac{\partial \tilde{p}}{\partial t} = c_0^2 \frac{\partial \tilde{\rho}}{\partial t}$

idem $\vec{\nabla} \tilde{p} = c_0^2 \vec{\nabla} \tilde{\rho}$

⚠ pas de ~ (1)

CM:

Q13) $\frac{\text{Terme non linéaire}}{\text{Terme linéaire}} \sim \frac{\delta \rho \cancel{U} \cancel{X}}{\cancel{X} \rho_0 \cancel{U}} = \frac{\delta \rho}{\rho_0} = M.$

BQn.

$\frac{TNL}{TL} = \frac{\cancel{\rho_0} U^{\cancel{2}}}{\cancel{\rho_0} \cancel{U} f \lambda} = \frac{U}{c_0} = n.$ (1)

Q14) Combinaison ρ, p (1)

Vitesse (1)

Q15) Non car on a négligé les termes visqueux 0,5

Q16) Terme de dérivée de + gd ordre nul 0,5

Q17) Echelle caractéristique a.

$$\text{Cn} : \underbrace{\delta\rho_f \frac{\partial\overline{\rho}}{\partial\overline{t}}}_{(2)} + \underbrace{\frac{\rho_0 U}{a} \operatorname{div} \overline{v}}_{(1)} + \cancel{\frac{\delta\rho U}{a} \vec{v} \cdot \vec{\nabla} \overline{\rho}} = 0.$$

négligeable.

$$\frac{(1)}{(2)} \sim \frac{\rho_0 U}{a} \times \frac{1}{\delta\rho f} = \frac{\rho_0}{\delta\rho} \times \frac{U}{af}$$

$$= \frac{c_0}{U} \times \frac{1}{af}$$

$$= \frac{\lambda}{a} \gg 1.$$

Donc localement $\operatorname{div} \vec{v} = 0$.

Q18)