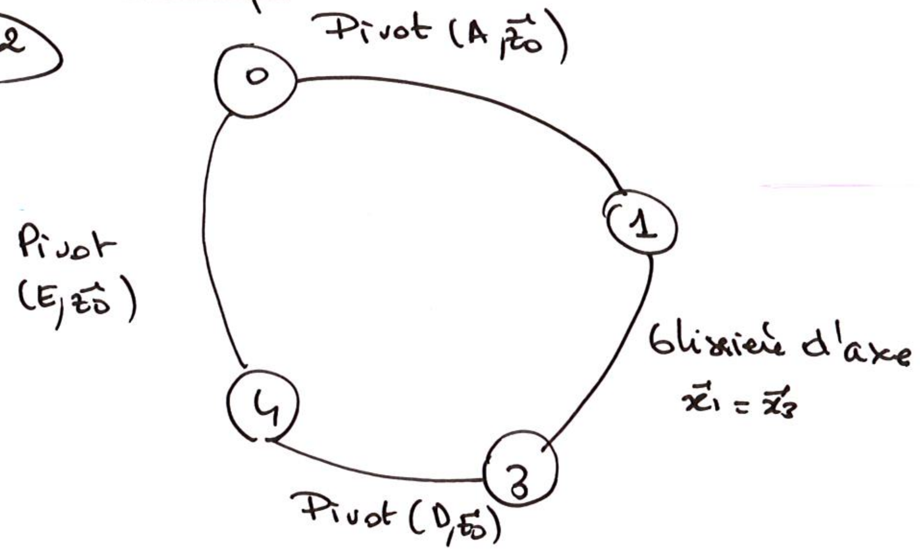
Corrigé DS 2 N & I
2022 - 2023

1 Question de cours

Q1) Cf cours (2,5) points.

2.1 Cinématique

Q2)



(0,5)

Q3) $h = 6\gamma + m - I_c$

$\gamma = 1$ (cf graphe des liaisons)

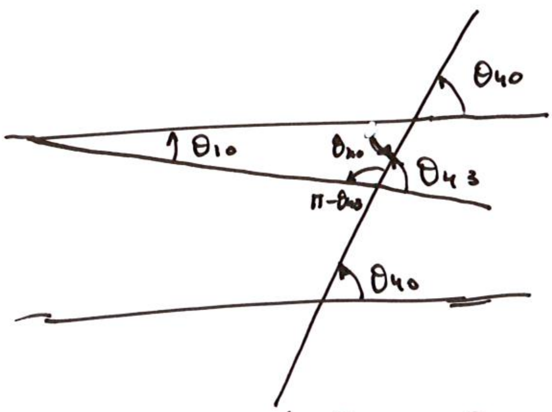
$m = 1$

$I_c = 1 + 1 + 1 + 1 = 4.$

(1)

$h = 6 + 1 - 4 = 3$

Q4)



$\theta_{10} + \theta_{40} + (\cancel{\pi} - \theta_{43}) = \cancel{\pi}$

$\boxed{\theta_{10} + \theta_{40} = \theta_{43}}$

Donc $\boxed{\dot{\theta}_{10} + \dot{\theta}_{40} = \dot{\theta}_{43}}$

(0,5)

Q5 $\{T_c(1/0)\} = \left\{ \begin{matrix} \dot{\theta}_{10}\,\vec{z_0} \\ \vec{0} \end{matrix} \right\}_A$ (0,5)

$\{T_c(3/1)\} = \left\{ \begin{matrix} \vec{0} \\ \dot{\lambda}_{31}\,\vec{x_1} \end{matrix} \right\}_{P\forall P}$ (0,5)

$\{T_c(4/3)\} = \left\{ \begin{matrix} \dot{\theta}_{43}\,\vec{z_0} \\ \vec{0} \end{matrix} \right\}_D$ (0,5)

$\{T_c(4/0)\} = \left\{ \begin{matrix} \dot{\theta}_{40}\,\vec{z_0} \\ \vec{0} \end{matrix} \right\}_E$ (0,5)

Q6 $\vec{V}(E \in 4/3) = \vec{V}(D \in 4/3) + \vec{\Omega}(4/3) \wedge \vec{DE}$

$= \dot{\theta}_{43}\,\vec{z_0} \wedge L_{41}\,\vec{x_4}$

$= \dot{\theta}_{43}\,L_{41}\,\vec{y_4}$ (1)

$\vec{V}(E \in 1/0) = \vec{V}(A \in 1/0) + \vec{\Omega}(1/0) \wedge \vec{AE}$

$= \dot{\theta}_{10}\,\vec{z_0} \wedge (\lambda_{31}\,\vec{x_1} - L_{41}\,\vec{x_4})$ (1)

$= \dot{\theta}_{10}\,\lambda_{31}\,\vec{y_1} - \dot{\theta}_{10}\,L_{41}\,\vec{y_4}$

$\vec{V}(E \in 3/1) = \dot{\lambda}_{31}\,\vec{x_1}$ (0,5)

Q7) $\begin{cases} \vec{x_1} = \cos\theta_{10}\,\vec{x_0} + \sin\theta_{10}\,\vec{y_0} \\ \vec{y_1} = -\sin\theta_{10}\,\vec{x_0} + \cos\theta_{10}\,\vec{y_0} \end{cases}$ 0,5

$\begin{cases} \vec{x_4} = \cos\theta_{40}\,\vec{x_0} + \sin\theta_{40}\,\vec{y_0} \\ \vec{y_4} = -\sin\theta_{40}\,\vec{x_0} + \cos\theta_{40}\,\vec{y_0} \end{cases}$ 0,5

Q8) Oui 2D car toutes les translations ds le plan $(A, \vec{x_0}, \vec{y_0})$ et toutes les rotations d'axe ⊥ au plan. 0,5

Inconnues du pb : θ_{10}, θ_{43}, θ_{40}, λ_{31}

Equations : vitesse 2

vitesse angulaire 1.

On a 4 inconnues, 3 équations et on cherche une relation entrée sortie donc oui bien posé 1

Q9) Composition des mvts :

$\{T_C(4/0)\} = \{T_C(4/3)\} + \{T_C(3/2)\} + \{T_C(2/1)\} + \{T_C(1/0)\}$

$\dot{\theta}_{40} = \dot{\theta}_{43} + \dot{\theta}_{10}$ 0,25

$\vec{0} = \dot{\theta}_{43} L_{41} \vec{y_4} + \dot{\lambda}_{31} \vec{x_1} + \dot{\theta}_{10} \lambda_{31} \vec{y_1} - \dot{\theta}_{10} L_{41} \vec{y_4}$.

0,75

Q10)

On projette sur $\vec{x_0}$ et $\vec{y_0}$

$$\begin{cases} \vec{x_0} : \dot{\theta}_{43} L_{41} \sin\theta_{43} + \dot{\lambda}_{31} \cos\theta_{10} - \dot{\theta}_{10} \lambda_{31} \sin\theta_{10} - \dot{\theta}_{10} L_{41} \sin\theta_{40} = \\ \vec{y_0} : \dot{\theta}_{43} L_{41} \cos\theta_{43} + \dot{\lambda}_{31} \sin\theta_{10} + \dot{\theta}_{10} \lambda_{31} \cos\theta_{10} - \dot{\theta}_{10} L_{41} \cos\theta_{40} = 0 \end{cases}$$

1

$$\begin{cases} (\dot{\theta}_{40} - \dot{\theta}_{10}) L_{41} \sin(\theta_{40} - \theta_{10}) + \dot{\lambda}_{31} \cos\theta_{10} - \dot{\theta}_{10} \lambda_{31} \sin\theta_{10} - \dot{\theta}_{10} L_{41} \sin\theta_{40} = 0 \\ (\dot{\theta}_{40} - \dot{\theta}_{10}) L_{41} \cos(\theta_{40} - \theta_{10}) + \dot{\lambda}_{31} \sin\theta_{10} + \dot{\theta}_{10} \lambda_{31} \cos\theta_{10} - \dot{\theta}_{10} L_{41} \cos\theta_{40} = 0 \end{cases}$$

1

Il faudrait combiner ces 2 équations pour éliminer θ_{10} mais pas de solution simple.

2.2 Statique

Q10) $\vec{R} = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\int_{r=0}^{R} \rho_1 \vec{g}\, dV + \int_{\varphi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi}\int_{r=0}^{R} \rho_2 \vec{g}\, dV$

cst cst.

$$\boxed{\vec{R}(g\to 4) = \frac{2}{3}\pi R^3 (\rho_1 + \rho_2)\,\vec{g}}$$ (1)

$$\vec{M}_H(g\to 4) = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\int_{r=0}^{R} r\,\vec{e_r}\wedge \rho_1\vec{g}\; r\sin\theta\, dr\, d\theta\, d\varphi$$

$$+ \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\int_{r=0}^{R} r\,\vec{e_r}\wedge \rho_2\vec{g}\; r\sin\theta\, dr\, d\theta\, d\varphi.$$ (1)

$$\vec{e_r} = \sin\theta\cos\varphi\, \vec{z_0} + \sin\theta\sin\varphi\, \vec{x_4} + \cos\theta\, \vec{y_4}$$

$$\int_0^{2\pi} \cos\varphi\, d\varphi = 0 \qquad \int_0^{2\pi} \sin\varphi\, d\varphi = 0. \qquad \int_{\varphi=0}^{2\pi} d\varphi = 2\pi$$

$$\int_{\theta=0}^{\pi/2} \sin\theta\cos\theta\, d\theta = \left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2} = \frac{1}{2}$$

$$\int_{\pi/2}^{\pi} \sin\theta\cos\theta\, d\theta = \left[\frac{\sin^2\theta}{2}\right]_{\pi/2}^{\pi} = -\frac{1}{2}.$$

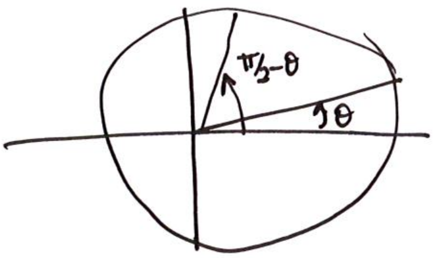
$$\int_0^{R} r^2\, dr = \frac{R^3}{3}$$

Donc il reste

$$\vec{M}_M(g \to 4) = \frac{\pi R^3}{3}\left(\vec{y_4} \wedge \rho_1 \vec{g} - \vec{y_4} \wedge \rho_2 \vec{g}\right) \quad 2$$

$$\vec{g} = \cos\left(\pi/2 - \theta_{40}\right) \vec{x_4} + \sin\left(\pi/2 - \theta_{40}\right) \vec{y_4}$$

Or



or $\cos\left(\pi/2 - \theta\right) = \sin\theta$

$\sin\left(\pi/2 - \theta\right) = \cos\theta$.

$$\vec{g} = g\left(\sin\theta_{40}\, \vec{x_4} + \cos(\theta_{40})\, \vec{y_4}\right)$$

$$\vec{y_4} \wedge \vec{g} = -\sin\theta_{40}\, g\, \vec{z_0}$$

$$\boxed{\vec{M}_M(\vec{g} \to 4) = \frac{\pi R^3}{3} \sin\theta_{40}\, g\, (\rho_2 - \rho_1)\, \vec{z_0}} \quad 1$$

Q11 $$\{F(0\to 1)\} = \left\{\begin{array}{c|c} X_{01} & \cancel{L_{01}} \\ Y_{01} & \cancel{M_{01}} \\ \cancel{Z_{01}} & 0 \end{array}\right\}_{A}^{R_0}$$

0,5

Pouvaient se placer en 2D.

$$\{F(3\to 4)\} = \left\{\begin{array}{c|c} X_{34} & \cancel{L_{34}} \\ Y_{34} & \cancel{M_{34}} \\ \cancel{Z_{34}} & 0 \end{array}\right\}_{D}^{R_0}$$

0,5

$$\{F(0\to 4)\} = \left\{\begin{array}{c|c} X_{04} & \cancel{L_{04}} \\ Y_{04} & \cancel{M_{04}} \\ \cancel{Z_{04}} & 0 \end{array}\right\}_{E}^{R_0}$$

0,5

$$\{F(1\to 3)\} = \left\{\begin{array}{c|c} X_{13} & \cancel{L_{13}} \\ Y_{13} & \cancel{M_{13}} \\ \cancel{Z_{13}} & N_{13} \end{array}\right\}_{C}^{R_0}$$

avec $\vec{R}(1\to 3) = X_{13}\,\vec{x_0} + Y_{13}\,\vec{y_0} + \cancel{Z_{13}\,\vec{z_0}}$

$= 0\,\vec{x_1} + Y_{13}^{(1)}\,\vec{y_1} + \cancel{Z_{13}\,\vec{z_0}}$

$\vec{x_1} = \cos\theta_{10}\,\vec{x_0} + \sin\theta_{10}\,\vec{y_0}$

$\vec{y_1} = -\sin\theta_{10}\,\vec{x_0} + \cos\theta_{10}\,\vec{y_0}$

Donc $X_{13}\cos\theta_{10} - Y_{13}\sin\theta_{10} = 0$

i.e $\boxed{X_{13} = \tan\theta_{0}\,Y_{13}}$

2

Q12) On isole 1

$$\{F(0\to1)\} + \{F(3\to1)\} + \{F(m\to1)\} = \{0\}.$$

En 2D, au point C, on obtient.

$$\vec{M_C}(0\to1) = \vec{M_A}(0\to1) + \vec{R}(0\to1) \wedge \vec{AC}$$ On note $AC = \lambda_{AC}$

$$= \vec{0} + \begin{vmatrix} X_{01} \\ Y_{01} \\ \times \end{vmatrix} \wedge \begin{vmatrix} \cos\theta_{10}\lambda_{AC} \\ \sin\theta_{10}\lambda_{AC} \\ \times \end{vmatrix}$$

$$= (X_{01}\sin\theta_{10} - Y_{01}\cos\theta_{10})\lambda_{AC} \ \vec{z_0}$$

$$\vec{R}(m\to1) = -F_m\cos\theta_{10}\,\vec{x_0} - F_m\sin\theta_{10}\,\vec{y_0}.$$

Au final :

$$\begin{cases} X_{01} - X_{13} - F_m\cos\theta_{10} = 0 \\ Y_{01} - Y_{13} - F_m\sin\theta_{10} = 0 \\ (X_{01}\sin\theta_{10} - Y_{01}\cos\theta_{10})\lambda_{AC} - N_{13} = 0 \\ X_{13} = \tan\theta_{10} Y_{13} \end{cases}$$

3

Q13)

On isole 3 et on écrit le PFS en C

$$\{F(1\to3)\} + \{F(4\to3)\} + \{F(m\to3)\} = \{0\}$$

$$\vec{M}_C(4\to3) = \underbrace{\cancel{\vec{M}_D(4\to3)}}_{\vec{0}} + \vec{R}(4\to3)\wedge\vec{DC}$$

On note $DC = d_{DC}$

$$\vec{M}_C(4\to3) = \begin{vmatrix} -X_{34} \\ -Y_{34} \\ 0 \end{vmatrix} \wedge \begin{vmatrix} -\cos\theta_{10}\, d_{DC} \\ -\sin\theta_{10}\, d_{DC} \\ \end{vmatrix}$$

$$= (X_{34}\sin\theta_{10} - Y_{34}\cos\theta_{10})\, d_{DC}\, \vec{z_0}.$$

Au final :

$$\begin{cases} X_{13} - X_{34} + F_m\cos\theta_{10} = 0 \\ Y_{13} - Y_{34} + F_m\sin\theta_{10} = 0 \\ N_{13} + (X_{34}\sin\theta_{10} - Y_{34}\cos\theta_{10})\, d_{DC} = 0 \\ X_{13} = \tan\theta_{10}\, Y_{13} \end{cases}$$

3

Q14) On isole 4 et on écrit le PFS en M D ou E

$$\{F(3\to4)\} + \{F(g\to4)\} + \{F(0\to4)\} = \{0\}$$

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