Corrigé DS2

M&I

2023-2024

1. Exercice de cours 1

Q1.1) $\{T_c(S_1/R_0)\} = \begin{Bmatrix} \omega_1 \vec{z_0} \\ \vec{0} \end{Bmatrix}_O$

$\{T_c(S_2/R_0)\} = \begin{Bmatrix} \omega_2 \vec{z_0} \\ \vec{0} \end{Bmatrix}_B$

$\boxed{1}$

$$\vec{V}(A\in S_1/R_0) = \underbrace{\cancel{\vec{V}(O\in S_1/R_0)}}_{\vec{0}} + \overrightarrow{\Omega(S_1/R_0)} \wedge \overrightarrow{OA}$$

$$= \omega_1 \vec{z_0} \wedge R_1 \vec{y_0}$$

$$= -\omega_1 R_1 \vec{x_0}$$

$$\vec{V}(A\in S_2/R_0) = \underbrace{\cancel{\vec{V}(B\in S_2/R_0)}}_{\vec{0}} + \vec{\Omega}(S_2/R_0) \wedge \overrightarrow{BA}$$

$$= \omega_2 \vec{z_0} \wedge R_2 \vec{y_0}$$

$$= -\omega_2 R_2 \vec{x_0}$$

$\boxed{1}$

Condition d'adhérence :

$$\vec{V}(A\in S_2/S_1) = \vec{0}$$

ie $\vec{V}(A\in S_2/R_0) = \vec{V}(A\in S_1/R_0)$

ie $-\omega_1 R_1 \vec{x_0} = -\omega_2 R_2 \vec{x_0}$ $\boxed{1}$

ie $\boxed{\dfrac{\omega_2}{\omega_1} = \dfrac{R_1}{R_2}}$ 3

2. Exercice de cours 2

Q2.1 $V = \displaystyle\int_{r=R_a}^{R_b} \int_{\theta=0}^{\pi/4} \int_{\varphi=0}^{\pi/2} r^2 \sin\theta \, d\theta \, d\varphi \, dr$.

$$= \left[\frac{r^3}{3}\right]_{R_A}^{R_b} \times \left[-\cos\theta\right]_0^{\pi/4} \times \left[\varphi\right]_0^{\pi/2}$$

$$= \left[\frac{R_b^3 - R_a^3}{3}\right] \times \left[1 - \frac{\sqrt{2}}{2}\right] \times \frac{\pi}{2}$$

1 point

Q.2.2 $\vec{R}(g \to C) = \displaystyle\int_{r=R_a}^{R_b} \int_{\theta=0}^{\pi/4} \int_{\varphi=0}^{\pi/4} -\rho_1 g \vec{z} \, r^2 \sin\theta \, d\theta \, d\varphi \, dr$

$$+ \int_{r=R_a}^{R_b} \int_{\theta=0}^{\pi/4} \int_{\varphi=\pi/4}^{\pi/2} -\rho_2 g \vec{z} \, r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$= -g\vec{z} \left[\frac{R_b^3 - R_a^3}{3}\right] \left[1 - \frac{\sqrt{2}}{2}\right] \times \frac{\pi}{4} \left[\rho_1 + \rho_2\right]$$

1 point

Q23) $\vec{M}_O(g \to C)$

$$= \int_{r=Ra}^{Rb} \int_{\theta=0}^{\pi/4} \int_{\varphi=0}^{\pi/4} r\,\vec{e_r} \wedge (-\rho_1 g\,\vec{z})\, r^2 \sin\theta\, d\theta\, d\varphi\, dr$$

$$+ \quad '' \quad '' \quad \int_{\varphi=\pi/4}^{\pi/2} r\,\vec{e_r} \wedge (-\rho_2 g\,\vec{z})\, r^2 \sin\theta\, d\theta\, d\varphi\, dr$$

1

Intégrales à calculer :

$$\int_{Ra}^{Rb} r^3\, dr = \left[\frac{Rb^4 - Ra^4}{4}\right]$$

0,5

$$\vec{e_r} = \sin\theta\cos\varphi\,\vec{x} + \sin\theta\sin\varphi\,\vec{y} + \cos\theta\,\vec{z}.$$

$$\vec{e_r} \wedge \vec{z} = -\sin\theta\cos\varphi\,\vec{y} + \sin\theta\sin\varphi\,\vec{x}.$$

$$\int_{\theta=0}^{\pi/4} \sin^2\theta\, d\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta.$$
$$\Rightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/4}$$

1

$$= \left[\frac{\pi}{8} - \frac{1}{4}\right] = \frac{1}{8}\left[\pi - 2\right]$$

$$\int_0^{\pi/4} \cos\varphi\, d\varphi = \frac{\sqrt{2}}{2} \qquad \int_{\pi/4}^{\pi/2} \cos\varphi\, d\varphi = \left(1 - \frac{\sqrt{2}}{2}\right)$$

1,5

$$\int_0^{\pi/4} \sin\varphi\, d\varphi = \left(1 - \frac{\sqrt{2}}{2}\right) \qquad \int_{\pi/4}^{\pi/2} \sin\varphi\, d\varphi = \frac{\sqrt{2}}{2}$$

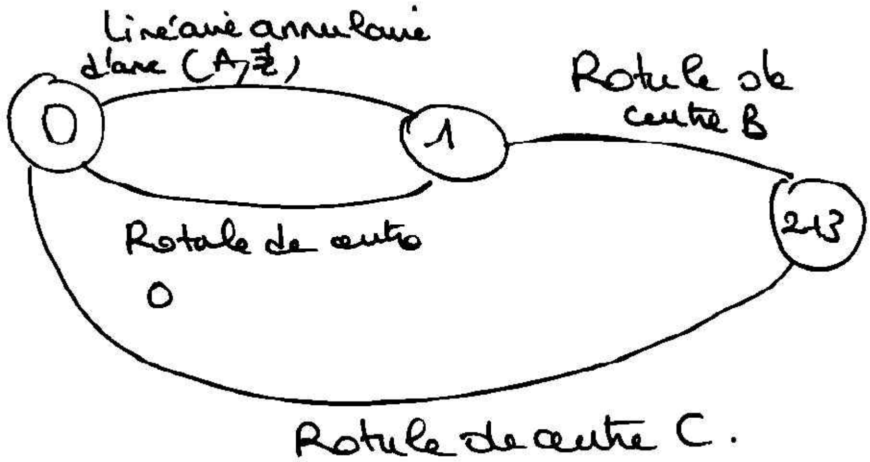
$$\vec{M}_O(g \to C)$$

$$= g\left[\frac{Rb^4 - Ra^4}{4}\right] \times \frac{1}{8}\left[\pi - 2\right] \times \left[\rho_1\left(\frac{\sqrt{2}}{2}\vec{x} - \left(1 - \frac{\sqrt{2}}{2}\right)\vec{y}\right) + \rho_2\left(\left(1 - \frac{\sqrt{2}}{2}\right)\vec{x} + \frac{\sqrt{2}}{2}\vec{y}\right)\right]$$

1

3 Problème de statique

Q3.1



0,5

Q3.2 Problème 3D car résultantes des 3 directions.

0,5

Q3.3

$$\{F(0^A \to 1)\} = \left\{\begin{array}{c|c} X_{01}^A & 0 \\ Y_{01}^A & 0 \\ 0 & 0 \end{array}\right\}_A$$

$$\{F(0^O \to 1)\} = \left\{\begin{array}{c|c} X_{01}^O & 0 \\ Y_{01}^O & 0 \\ Z_{01}^O & 0 \end{array}\right\}_O$$

2

$$\{F(2 \to 1)\} = \left\{\begin{array}{c|c} X_{21} & 0 \\ Y_{21} & 0 \\ Z_{21} & 0 \end{array}\right\}_B$$

$$\{F(0 \to 3)\} = \left\{\begin{array}{c|c} X_{03} & 0 \\ Y_{03} & 0 \\ Z_{03} & 0 \end{array}\right\}_C .$$

Q3.4) $\{F(eau \to 1)\} = \left\{ \begin{matrix} -F & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \right\}_P$ 1

$\{F(g \to 1)\} = \left\{ \begin{matrix} 0 & 0 \\ 0 & 0 \\ -mg & 0 \end{matrix} \right\}_G .$

Q3.5) Isolons 1 et appliquons le PFS :

$\{F(0^A \to 1)\} + \{F(0^O \to 1)\} + \{F(2 \to 1)\} + \{F(eau \to 1)\}$

$+ \{F(g \to 1)\} = \{0\}$.

Résultante :

$$\boxed{\begin{cases} X_{01}^A + X_{01}^O + X_{21} - F = 0 \\ Y_{01}^A + Y_{01}^O + Y_{21} = 0 \\ Z_{01}^O + Z_{21} - mg = 0 \end{cases}}$$ 1

Moments :

$\vec{M}_O(0^A \to 1) = \cancel{\vec{M}_A(0^A \to 1)}^{\vec{0}} + \vec{R}(0^A \to 1) \wedge \vec{AO}$

$= \begin{vmatrix} X_{01}^A \\ Y_{01}^A \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ 0 \\ L \end{vmatrix} = \begin{vmatrix} Y_{01}^A L \\ -X_{01}^A L \\ 0 \end{vmatrix}$

$$\vec{M}_O(2\to1) = \overset{\vec{0}}{\cancel{\vec{M}_B(2\to1)}} + \vec{R}(2\to1)\wedge\vec{BO}$$

$$= \left|\begin{matrix} X_{21} \\ Y_{21} \\ Z_{21} \end{matrix}\right. \wedge \left|\begin{matrix} 0 \\ -h \\ 0 \end{matrix}\right. = \left|\begin{matrix} Z_{21}h \\ 0 \\ -X_{21}h \end{matrix}\right. .$$

$$\vec{M}_O(\text{eau}\to1) = \overset{\vec{0}}{\cancel{\vec{M}_P(\text{eau}\to1)}} + \vec{R}(\text{eau}\to1)\wedge\vec{PO}$$

$$= \left|\begin{matrix} -F \\ 0 \\ 0 \end{matrix}\right. \wedge \left|\begin{matrix} 0 \\ -a \\ b \end{matrix}\right. = \left|\begin{matrix} 0 \\ Fb \\ Fa \end{matrix}\right.$$

$$\vec{M}_O(g\to1) = \overset{\vec{0}}{\cancel{\vec{M}_G(g\to1)}} + \vec{R}(g\to1)\wedge\vec{GO}$$

$$= \left|\begin{matrix} 0 \\ 0 \\ -mg \end{matrix}\right. \wedge \left|\begin{matrix} 0 \\ -c \\ d \end{matrix}\right. = \left|\begin{matrix} -mgc \\ 0 \\ 0 \end{matrix}\right. .$$

Au final :

$$\boxed{\begin{cases} Y_{01}^A L + Z_{21}h - mgc = 0 \\ -X_{01}^A L + Fb = 0 \\ -X_{21}h + Fa = 0 . \end{cases}}$$

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Q3.6) Isolons (2+3) :

Résultante

$$\boxed{\begin{cases} -X_{21} + X_{03} = 0 \\ -Y_{21} + Y_{03} = 0 \\ -Z_{21} + Z_{03} = 0 \end{cases}}$$

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Moment

$$\vec{M}_B(B \to 3) = \cancel{\vec{M}_C(C \to 3)}^{\vec{0}} + \vec{R}(0 \to 3) \wedge \vec{CB}$$

$$= \begin{vmatrix} X_{03} \\ Y_{03} \\ Z_{03} \end{vmatrix} \wedge \begin{vmatrix} \lambda \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ Z_{03}\lambda \\ -Y_{03}\lambda \end{vmatrix}$$

Au final :

$$\begin{cases} 0 = 0 \\ Z_{03} = 0 \\ Y_{03} = 0 \end{cases}$$

1

Q3.7)

Résolution :

$$\text{Q3.6} \Rightarrow \begin{cases} \boxed{Z_{03} = 0} \\ \boxed{Y_{03} = 0} \end{cases} \Rightarrow \begin{cases} \boxed{Y_{21} = 0} \\ \boxed{Z_{21} = 0} \end{cases} \qquad X_{21} = X_{03}$$

Il reste :

$$\begin{cases} X_{01}^{A} + X_{01}^{O} + X_{21} - F = 0 \\ Y_{01}^{A} + Y_{01}^{O} = 0 \\ Z_{01}^{O} - mg = 0 \end{cases}$$

$$\begin{cases} Y_{01}^{A} L - mgc = 0 \\ -X_{01}^{A} L + Fb = 0 \\ -X_{21} h + Fa = 0 \end{cases}$$

$$X_{21} = X_{03} = \frac{Fa}{h}$$

$$X_{01}^{A} = \frac{Fb}{L}$$

$$Y_{01}^{A} = \frac{mgc}{L} = -Y_{01}^{O}$$

$$Z_{01}^{O} = mg.$$

$$X_{01}^{O} = -X_{21} + F - X_{01}^{A} = -\frac{Fa}{h} + F - \frac{Fb}{L}$$

$$= F\left(1 - \frac{a}{h} - \frac{b}{L}\right)$$

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