

Energy Harvesting from Ambient Vibrations

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 - SSHI: Synchronized Switch Harvesting on Inductor

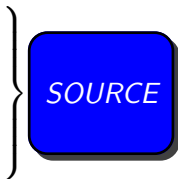
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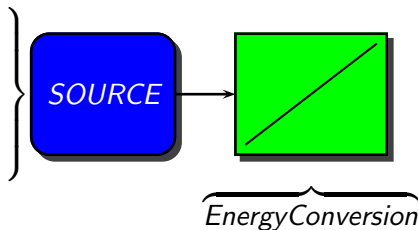
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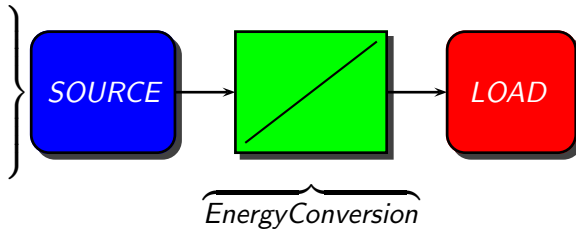
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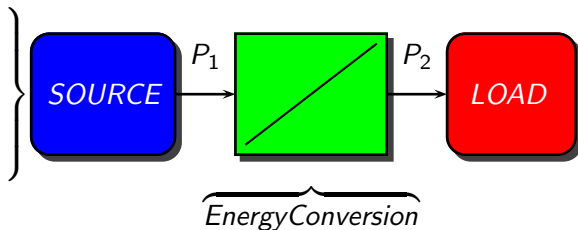
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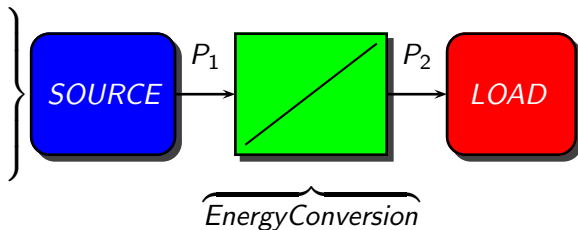
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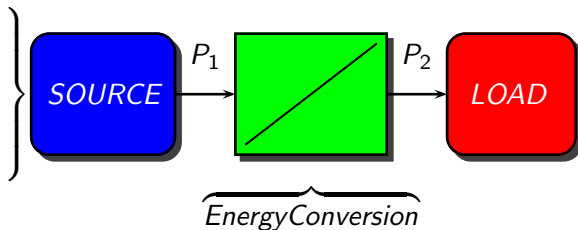


We talk about **Energy Harvesting** or also **energy scavenging** when the converted power is small, typically less than 1W.

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$\eta = \frac{P_2}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \rightarrow$ Losses in the energy converter should be as small as possible.

Objectives: Sensors Network

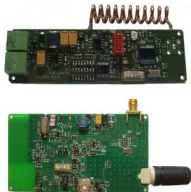


www.perpetuum.com

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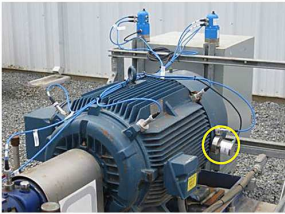


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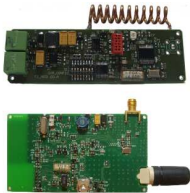


Gutiérrez, A Heterogeneous Wireless Identification Network for the Localization of Animals Based on Stochastic Movements

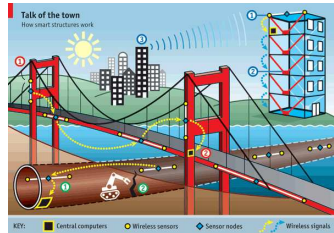
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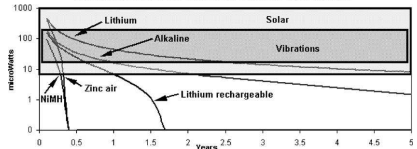


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<http://www.rfwirelessensors.com>, 2012

Continuous Power / cm² vs. Life for Several Power Sources



Roundy et Al.: A study of low level vibrations as a power source for wireless sensor nodes.

Objectives: Power, just where you need it



<http://enocean.com>

- Wireless
- Reduce Cost, and is reconfigurable
- Better Waste Cycle (Information from EnOcean)

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Innowattech's systems produces power with vehicles



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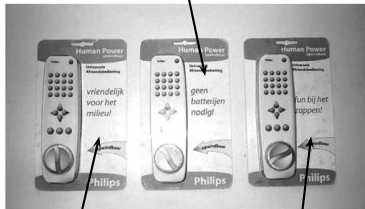


The economist – April 28th 2007

Objectives: Marketing Purpose

Experience: Same Remote controller energized by human power, but with different packaging.

"No need for Batteries"



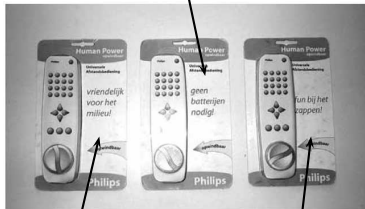
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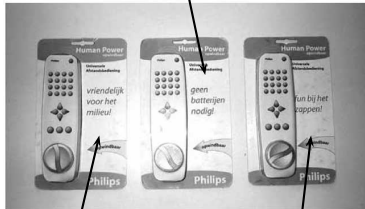
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54% of people Will choose the first one because it is eco-friendly.
(Jansen, Human power empirically explored)

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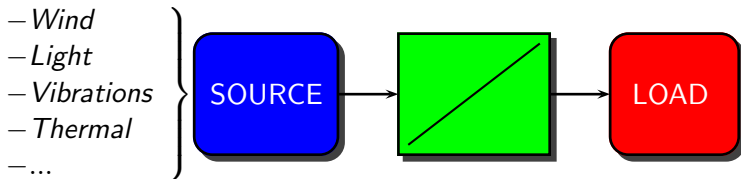
Several projects are born from this fact: **Metis** Produces energy from dancers' movements.



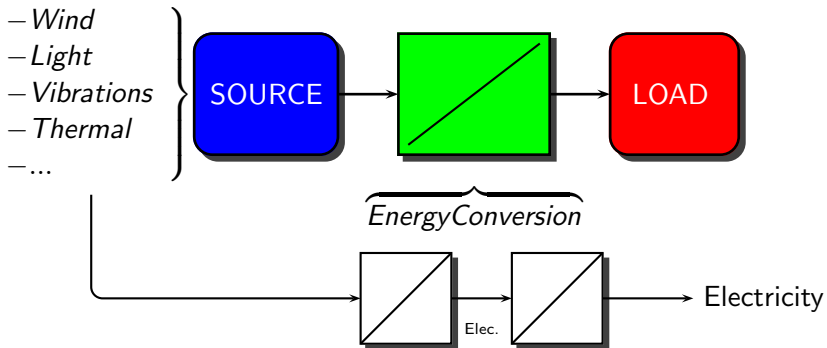
In Toulouse, the system **VIHA** proposes Smart Tiles to energies street lights.



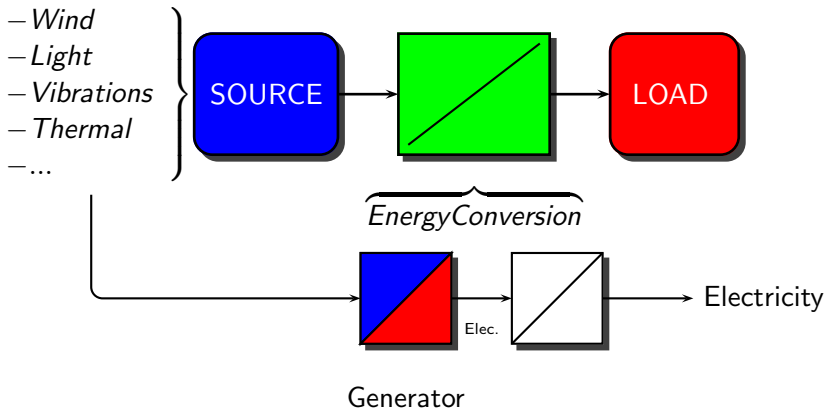
The energy converter



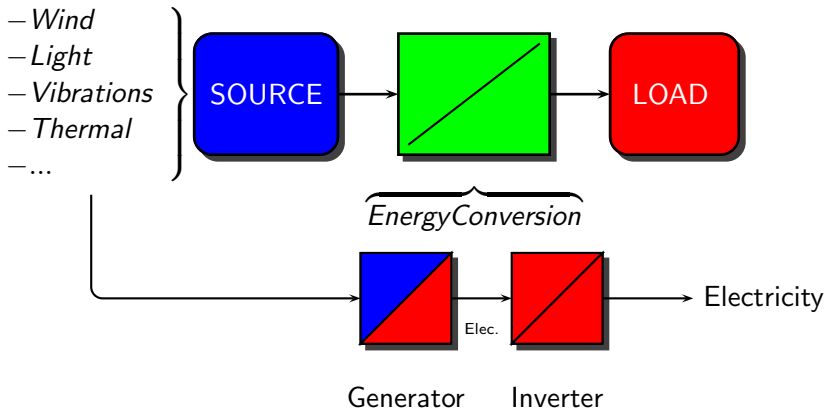
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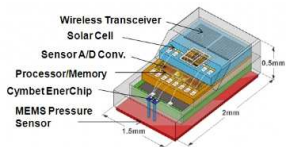


The energy converter



Solar Harvesters:

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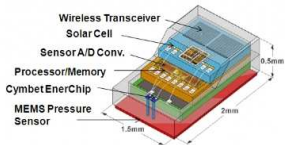


This sensor measures IntraOcular Pressure <http://cymbet.com>



Handbag to recharge electronic devices <http://www.neubers.de>

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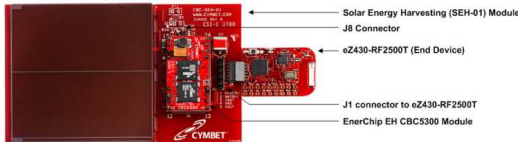
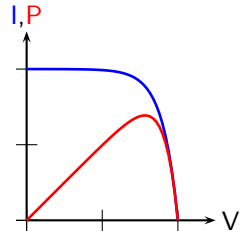


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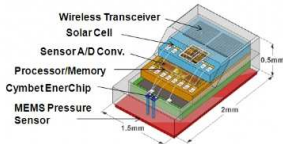
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Power and current as a function of voltage:



Solar Energy Harvester Evaluation Kit <http://ti.com>

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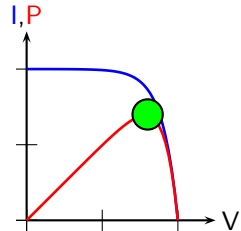


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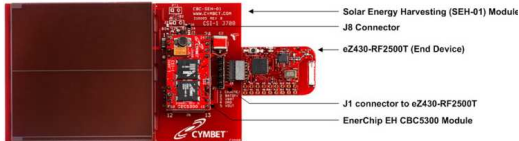


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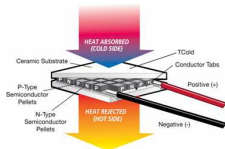


MPPT strategies, require Energy management of the system

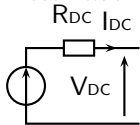


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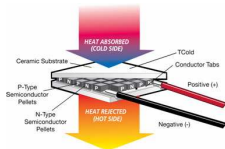
Thermoelectric



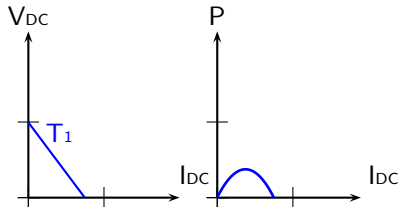
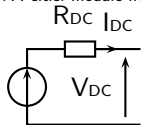
A Peltier module from <http://www.tellurex.com>



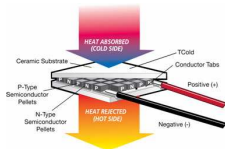
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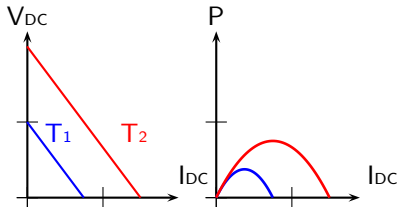
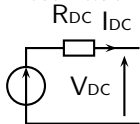
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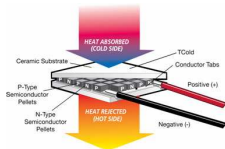
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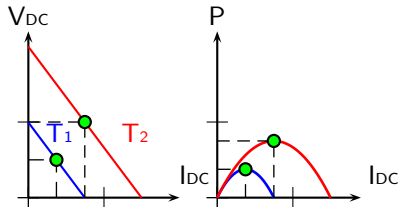
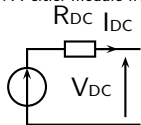
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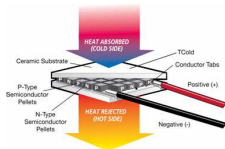
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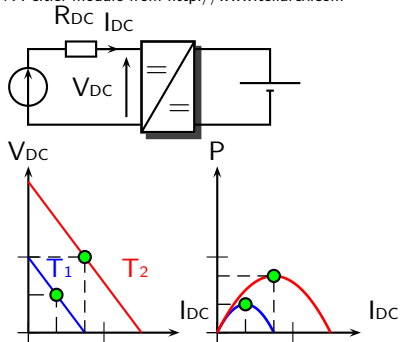
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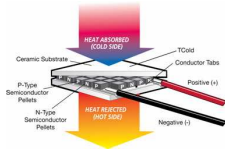
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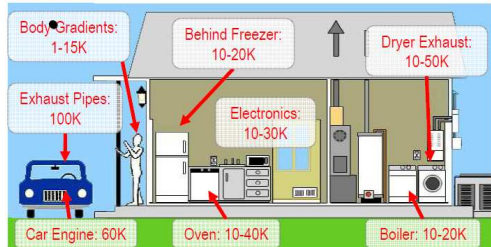
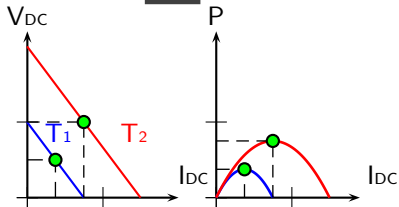
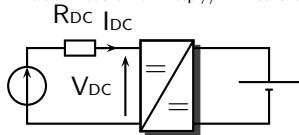
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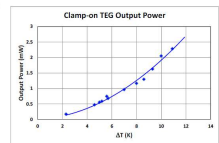


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Temperature Gradient (residential). Lindsay Miller,
<http://uc-ciee.org>

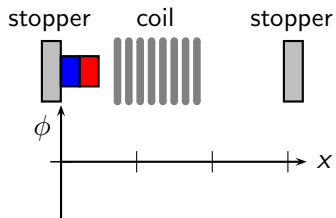
Clamp-on Drain Harvester



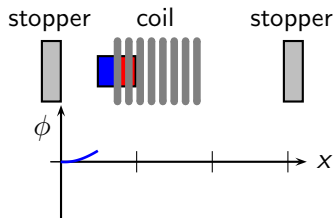
| ΔT | V_{load} (mV) | P_{max} (mW) |
|------------|-----------------|----------------|
| 5.0 | 38 | 0.56 |
| 8.0 | 55 | 1.16 |
| 10.0 | 73 | 2.05 |

plumbing application from <http://www.nextreme.com>

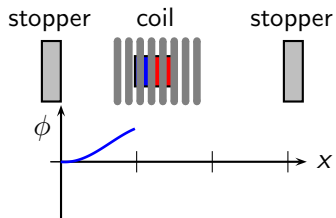
Magnetic



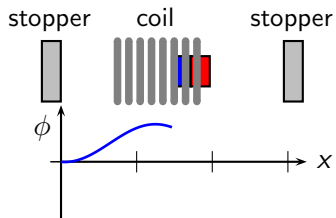
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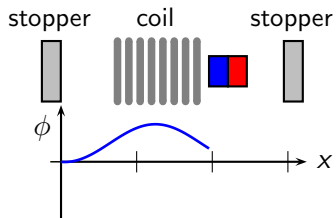
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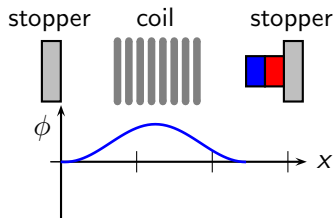
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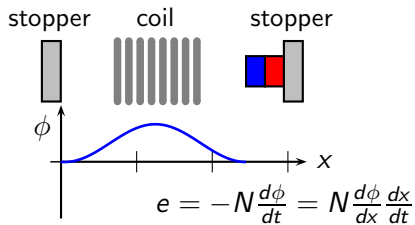
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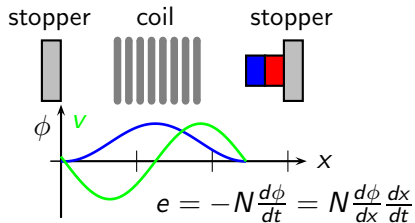
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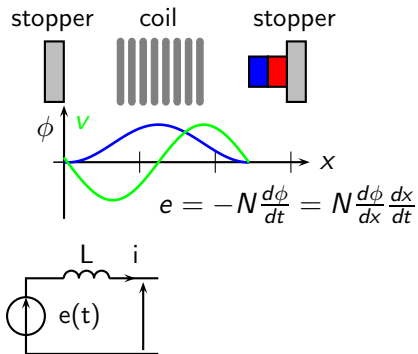
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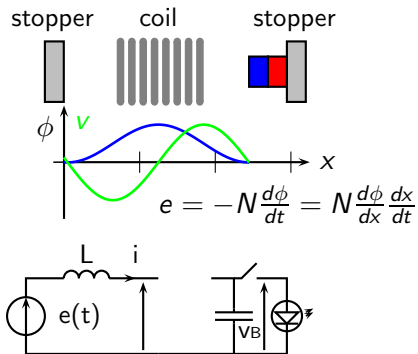
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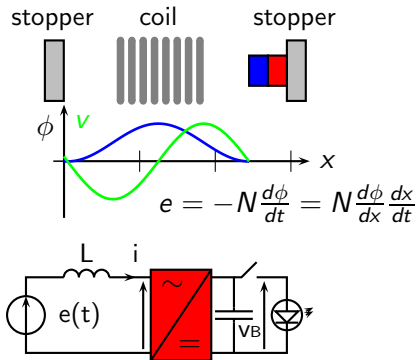
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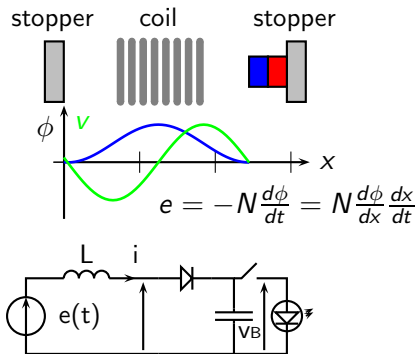
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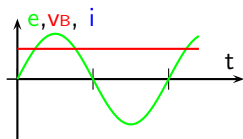
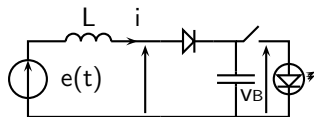
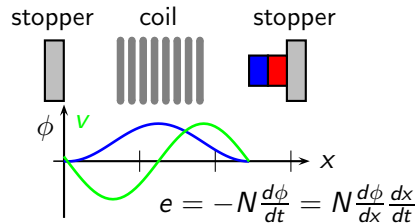
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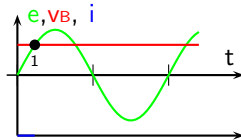
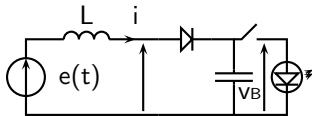
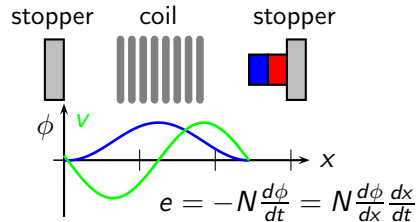
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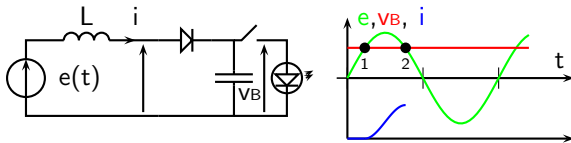
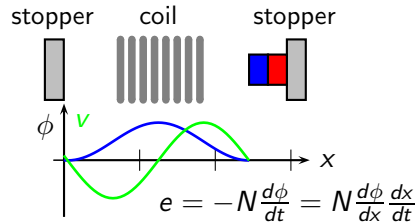


Magnetic



1) Diode turns ON

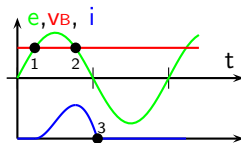
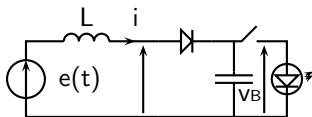
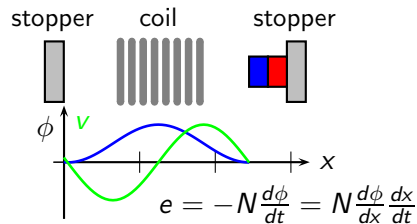
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- 2) $\frac{di}{dt} = 0$ because $e - v_B = 0$

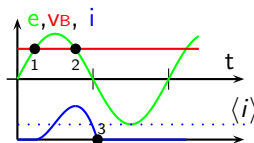
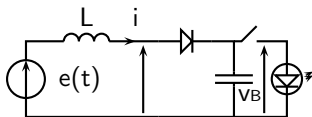
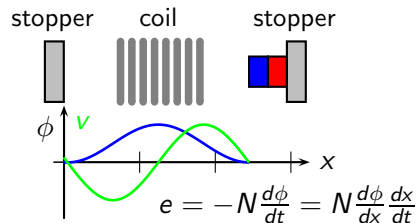


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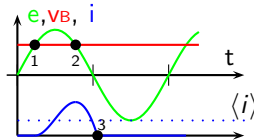
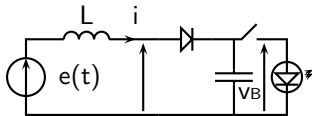
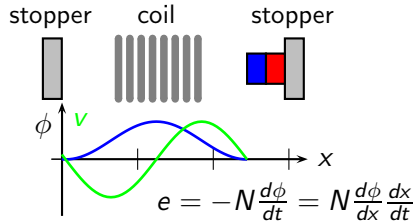
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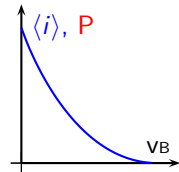


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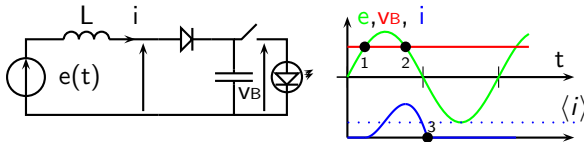
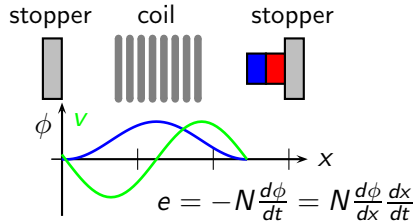
Magnetic



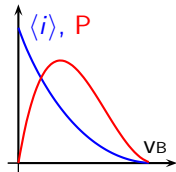
- 1) Diode turns ON
- 2) $\frac{di}{dt} = 0$ because $e - v_B = 0$
- 3) $i = 0$, diode turns OFF



Magnetic

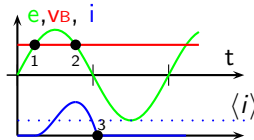
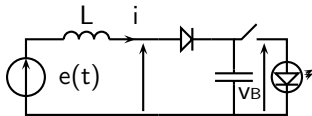
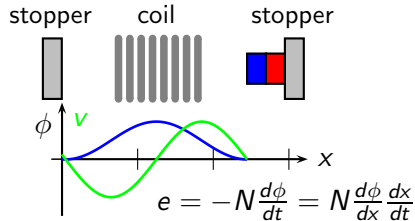


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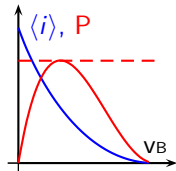


$$P = \langle v_B i \rangle = v_B \langle i \rangle$$

Magnetic

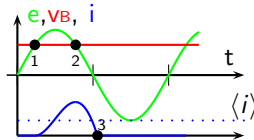
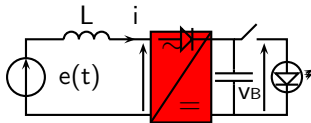
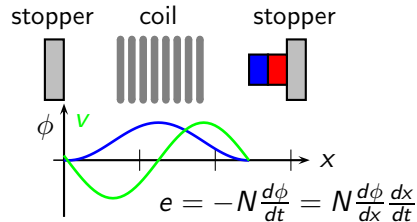


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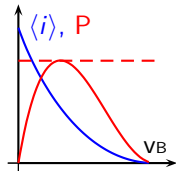


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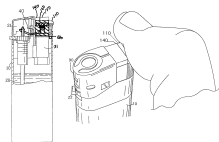


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Piezoelectric

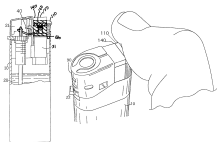


An electronic lighter,
<http://freepatentsonline.com>



Piezoelectric crystals

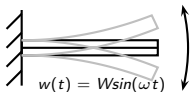
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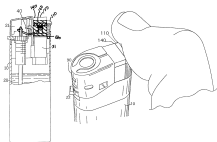
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Piezoelectric crystals
Cantilever beam



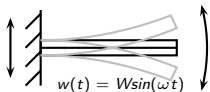
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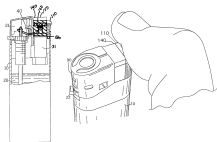
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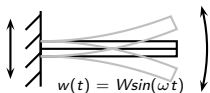


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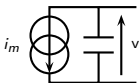


Piezoelectric crystals

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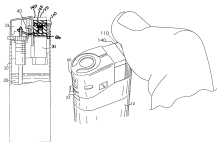


Equivalent electrical circuit



i_m is a current proportional
to the deformation speed

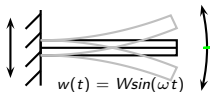
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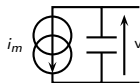
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Piezoelectric crystals
Cantilever beam

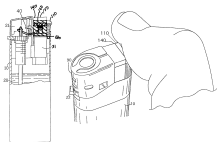


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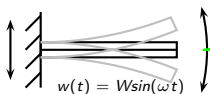
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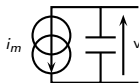
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Piezoelectric crystals
Cantilever beam



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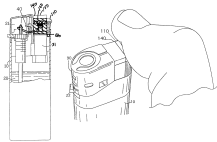


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Comparison Magn. Piezo

| | |
|----------------|----------------|
| Magnetic | Piezo. |
| Voltage source | Current source |
| Inductive | capacitive |
| Large Stroke | Small Stroke |
| Remote action | |

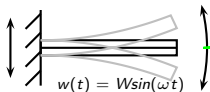
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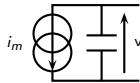
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Piezoelectric crystals
Cantilever beam

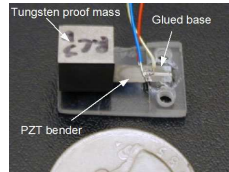


Equivalent electrical circuit



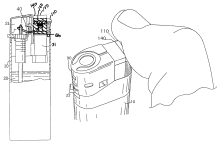
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)

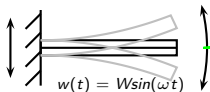
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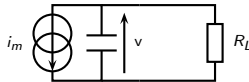
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Piezoelectric crystals
Cantilever beam

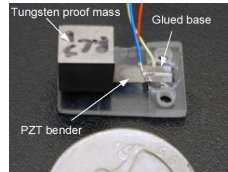


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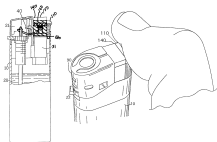
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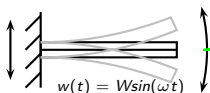
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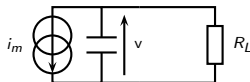
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Piezoelectric crystals
Cantilever beam

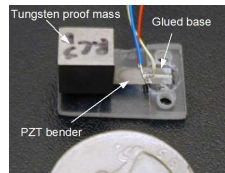


Equivalent electrical circuit

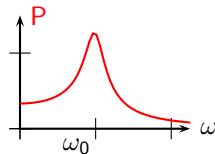


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Comparison Magn. Piezo

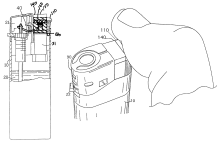
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



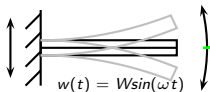
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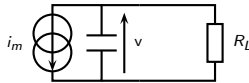
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Piezoelectric crystals
Cantilever beam

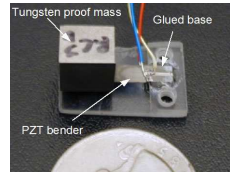


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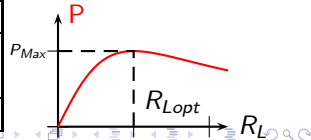
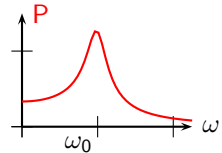


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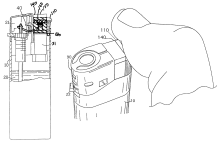
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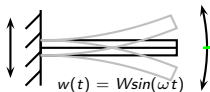
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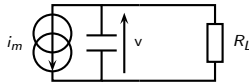


Piezoelectric crystals
Cantilever beam



Energy Management

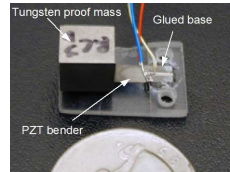
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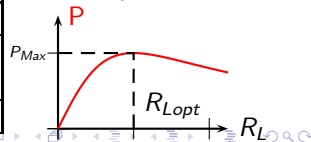
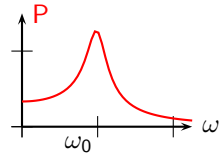
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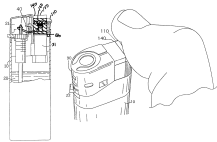
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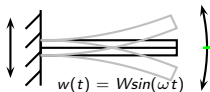
Piezoelectric



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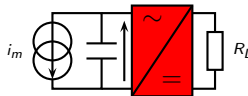


Piezoelectric crystals
Cantilever beam



Energy Management

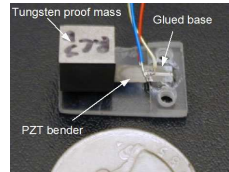
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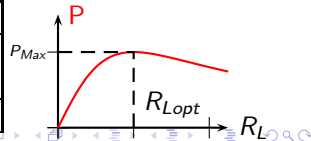
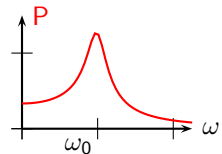
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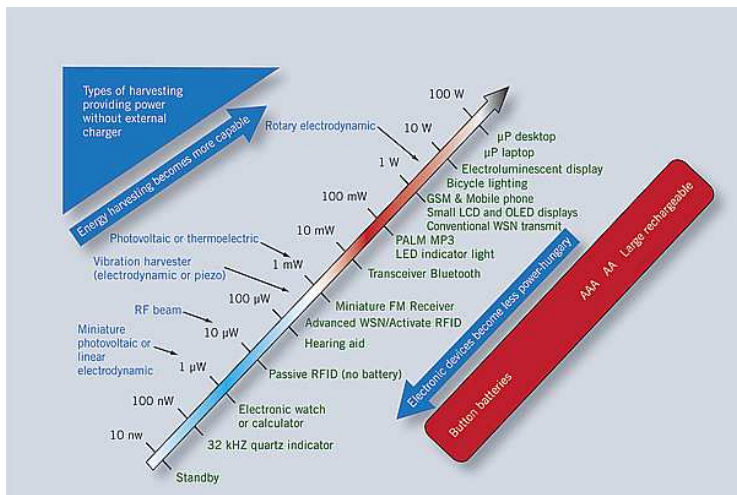
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Roundy, A piezoelectric vibration based generator for wireless electronics (2004)



There is no "one fit all" solution



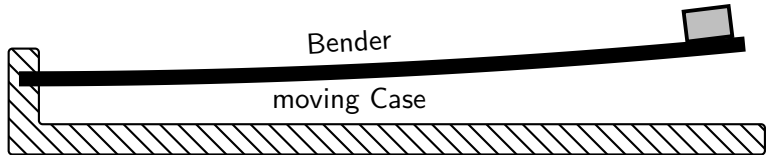
Each solution may be efficient in a certain range of Power.

Meanwhile, shrinking Chips consumption come at a time when energy harvesting becomes efficient and practical
(source: IDtechex.com)

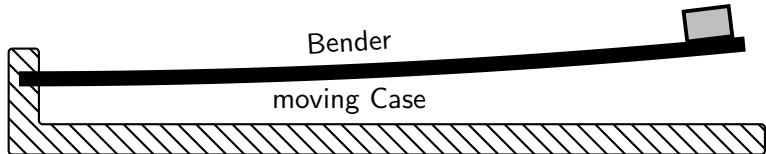
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 - Generator Technologies
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- 2 Modelling of a piezoelectric energy harvester
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 - EMR of the system
 - Power Extraction on a load resistor R_L from harmonic oscillation
- 3 An Example of inverter
 - Introduction
 - SSHI: Synchronized Switch Harvesting on Inductor

Coordinates and assumptions

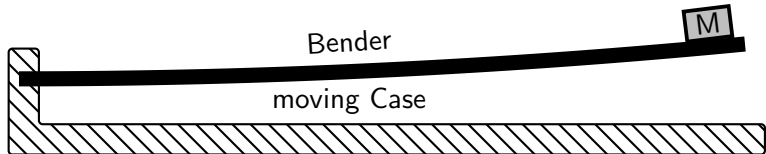


Coordinates and assumptions



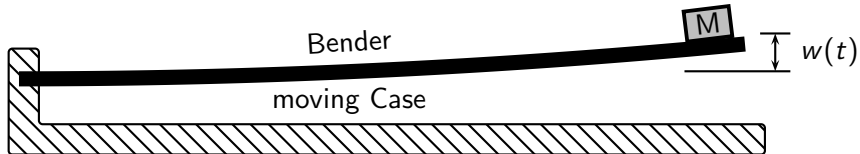
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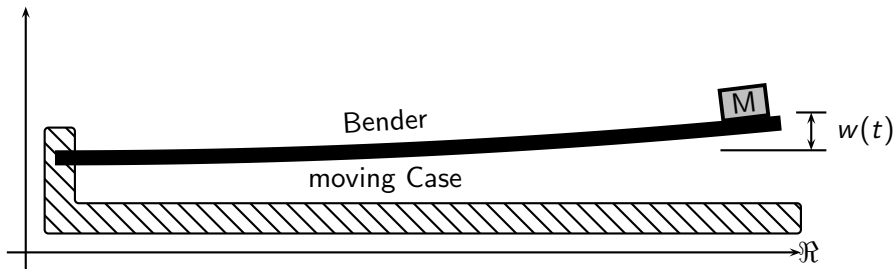
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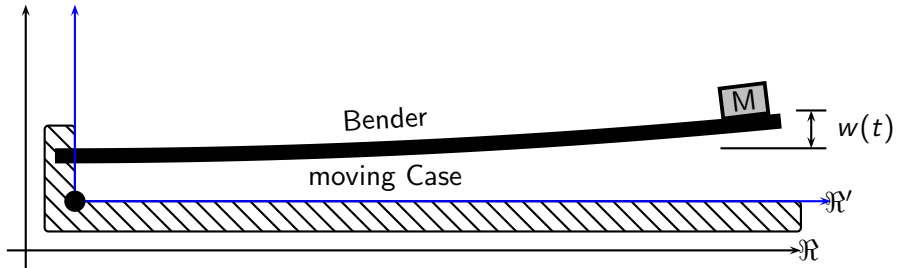
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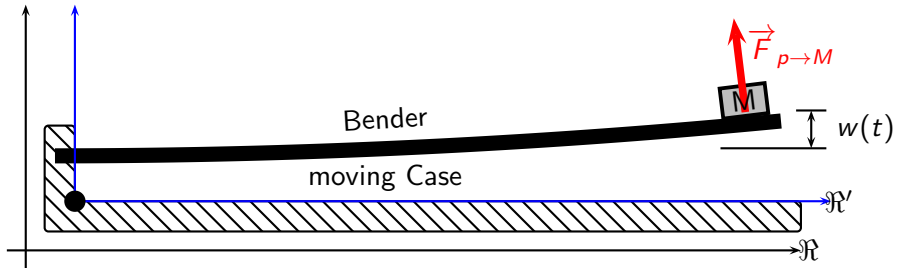
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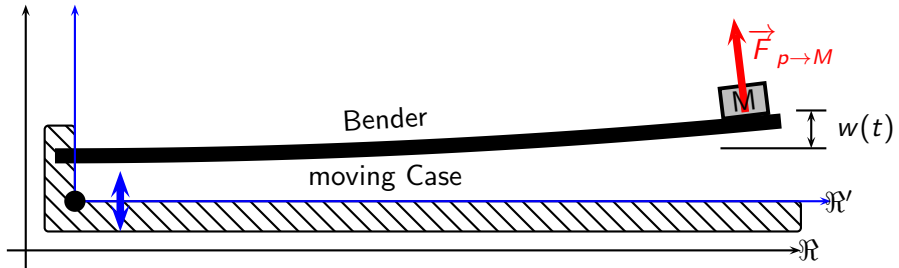
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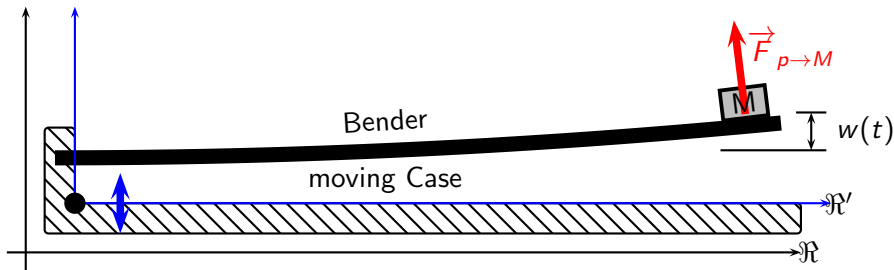
- $\vec{F}_{p \rightarrow m}$ is the force of the Bender onto the mass M .

Coordinates and assumptions



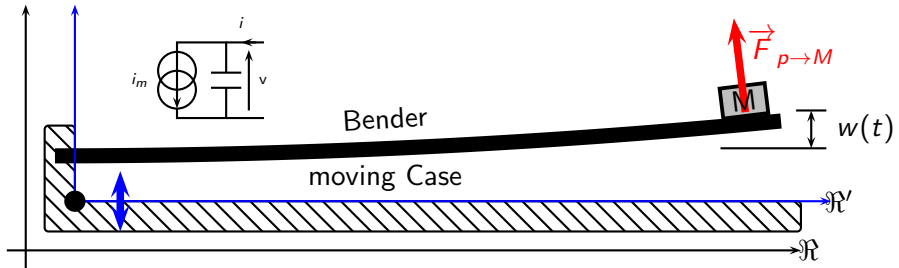
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- Gravity is neglected, as well as Inertia momentum of the bender,
- Actuator electrical convention.

Equations

- **Dynamic of the mass M :**

Glossary

- M the mass
- f , the force onto M
- A , vibration's amplitude
- ω , vibration's pulsation
- f_{acc} is the inertial force
- i current of the device (actuator convention)
- i_m motional current
- f_p inside piezo force
- N Piezoelectric force factor (depends on geometry)
- f_p Piezo internal force
- f_s Material internal elastic force
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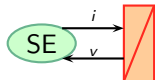
EMR of the system

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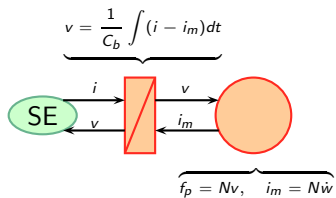


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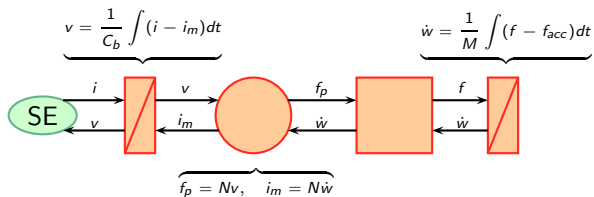
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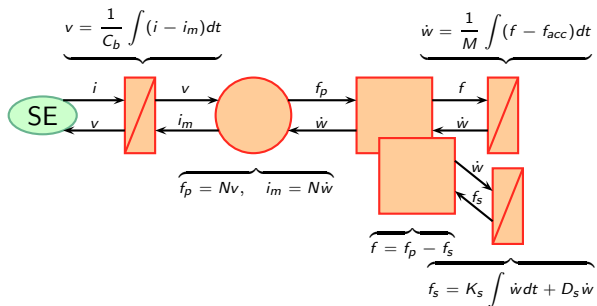
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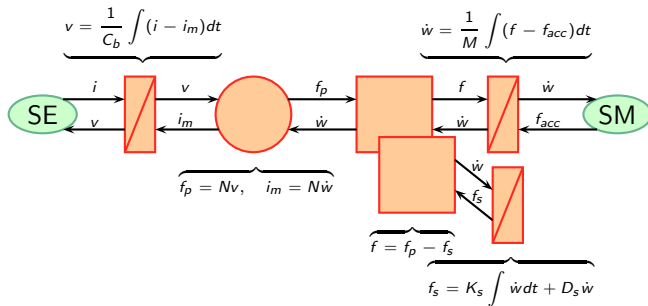
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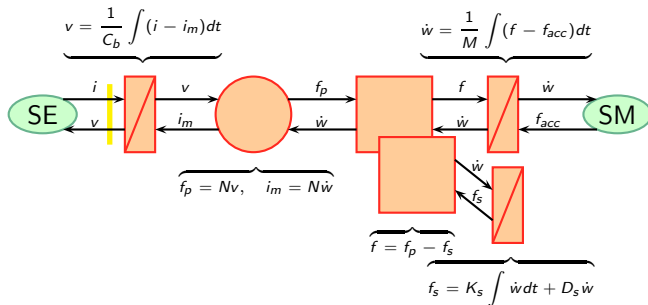
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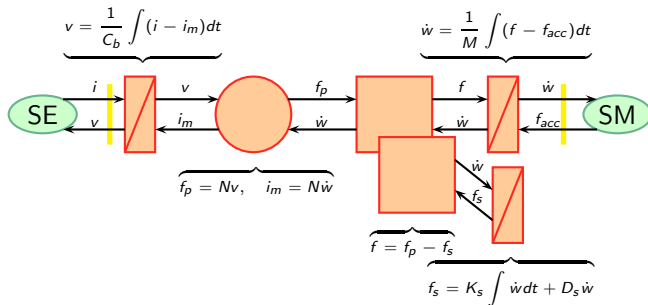


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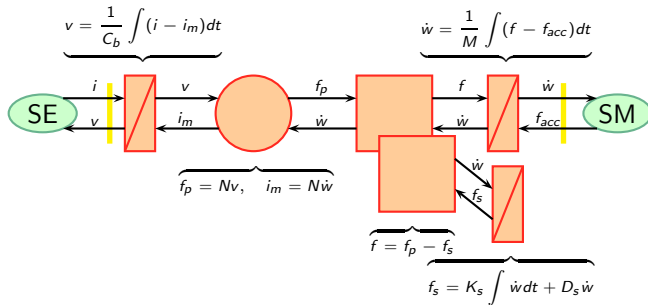
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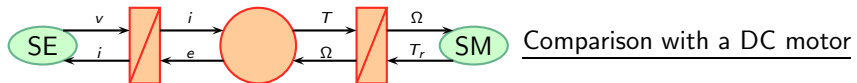


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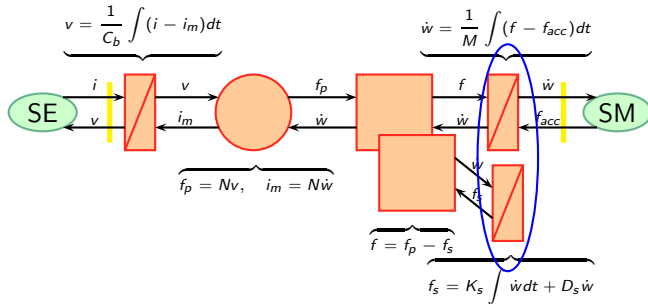


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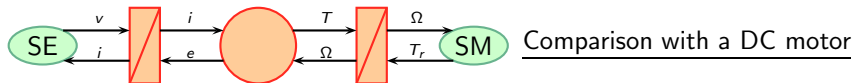


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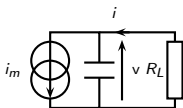
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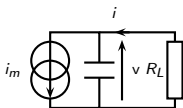
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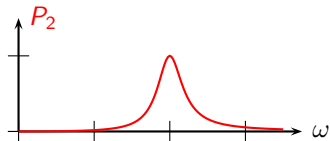
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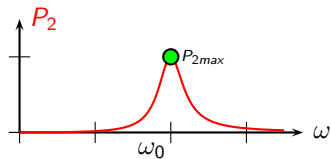
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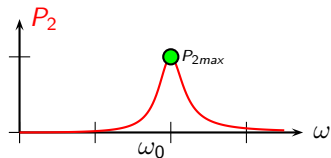
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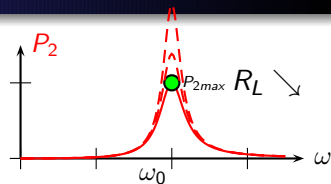


The vibrations should occur at generator's resonance frequency

$$\omega_0 = \sqrt{\frac{K_s}{M}}$$

For an ideal generator ($D_s = 0$)

- $M\ddot{w} = \underline{f} + \underline{f}_{acc}$
- $\underline{f} = N\underline{v} - K_s\underline{w}$
- $\underline{v} = -R_L\underline{i}_m = -R_L N\dot{w}$
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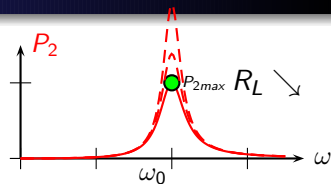


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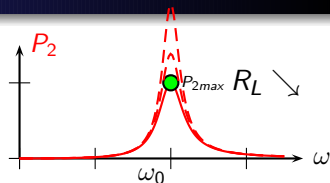


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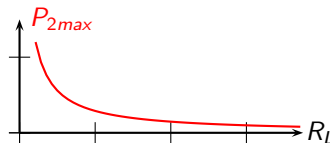
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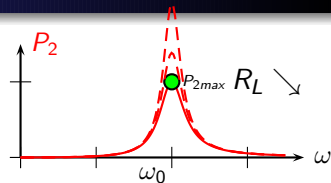
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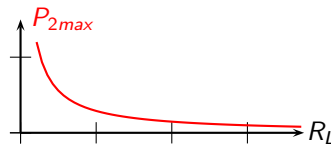
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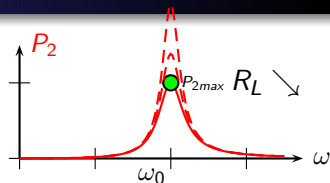
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We can harvest as much power as we want

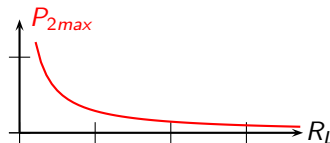
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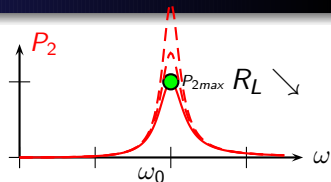
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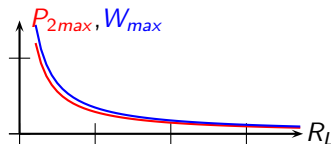
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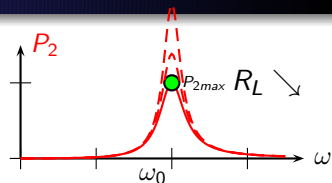
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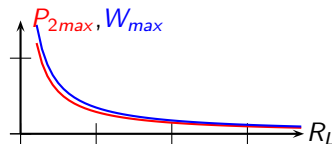
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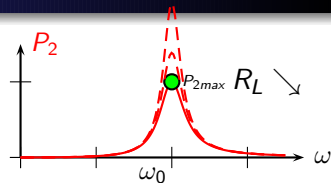
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We can harvest as much power as we want, at the expense of large displacement of the bender's tip

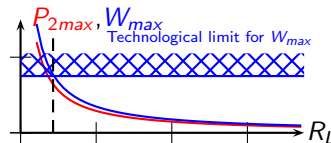
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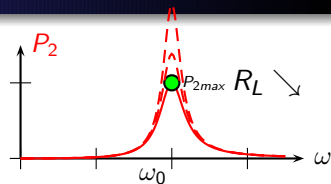
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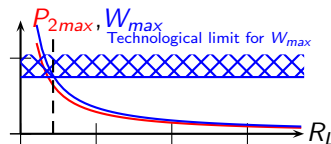
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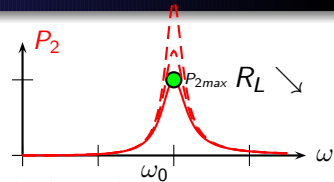
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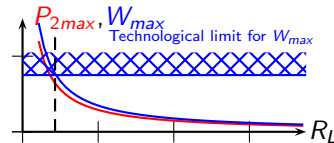
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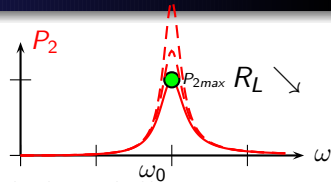
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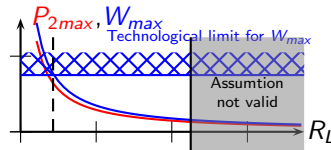
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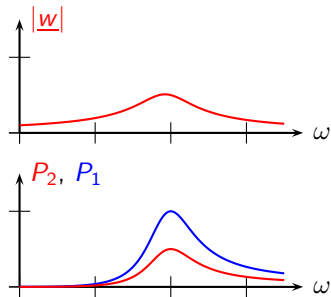
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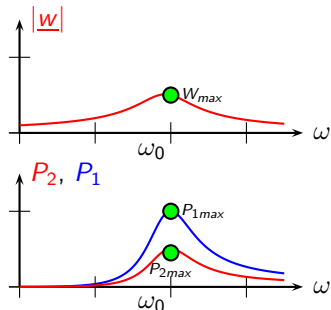
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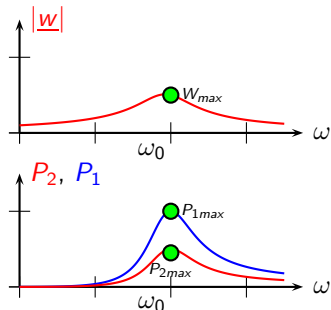
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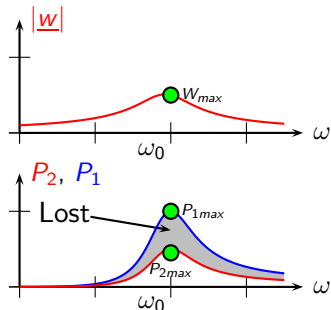
ω_0 should still be the vibration's pulsation.



For a real generator ($D_s \neq 0$)

- $\underline{f} = N\underline{v} - K_s\underline{w} - D_s\dot{\underline{w}}$
- $\underline{w} = \frac{\underline{f}_{acc}}{(K_s - M\omega^2) + j\omega(D_s + N^2R_L)}$
- $P_2 = -\frac{1}{2}R_L|i_m|^2$
- $P_2 = -\frac{1}{2} \frac{R_L N^2 \omega^2 |\underline{f}_{acc}|^2}{(K_s - M\omega^2)^2 + \omega^2 (D_s + N^2 R_L)^2}$
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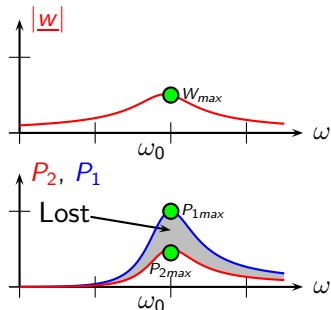
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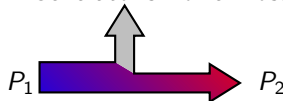
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Generator's Power Losses

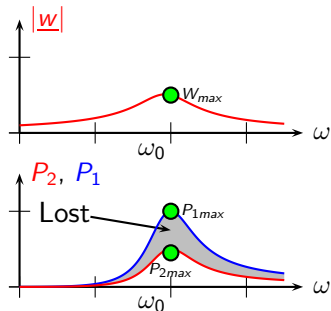


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Generator's Power Losses

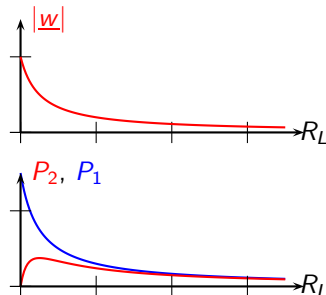


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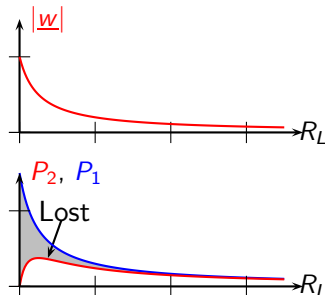
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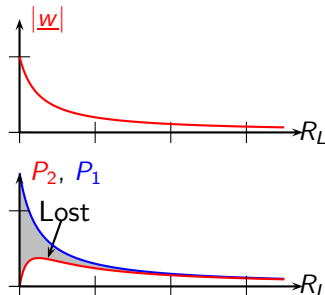


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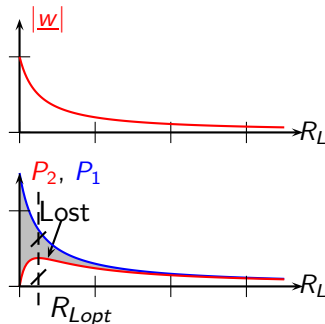


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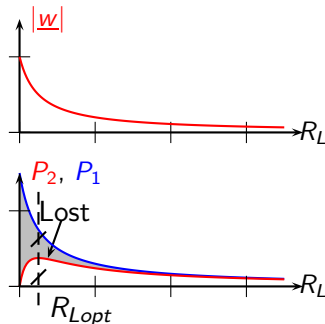


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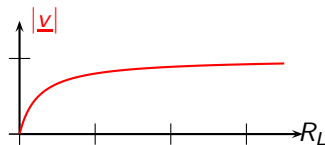
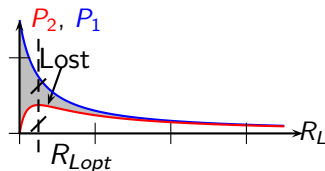
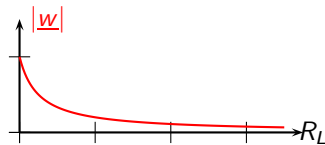


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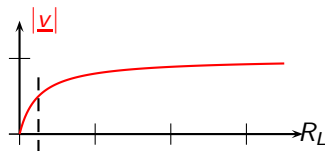
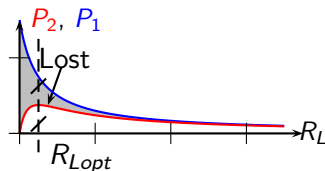
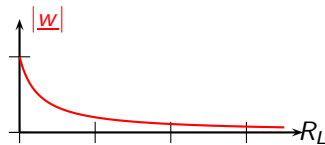


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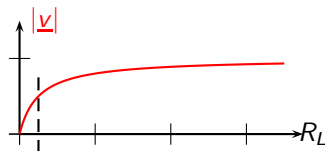
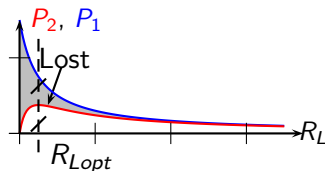
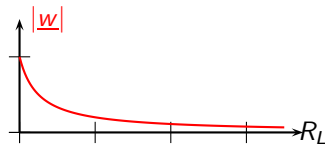


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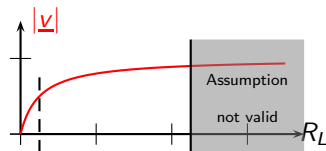
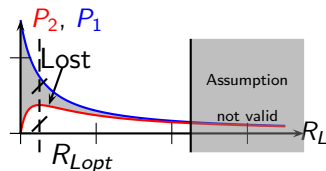
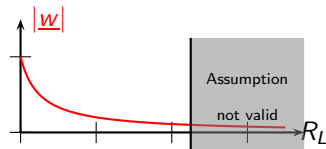


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Example

Device's properties

$$N = 0.012 \text{ N/V}, K_s = 6300 \text{ N/m}, C_b = 300 \text{ nF}, D_s = 0.17 \text{ Ns/m}, \\ M = 1 \text{ g}$$

Calculate for $A = 0.1 \text{ mm}$

- the best working frequency,
- the harvested P_2 power in the best case,
- the power of the source P_1 in such best case,
- the optimal resistor R_L ,
- the deflection amplitude of the bender,
- the voltage for this working point.

Validation

Is $v = -R_L i_m$ a valid assumption?

Answers

- The best working frequency is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}} = \frac{1}{2\pi} \sqrt{\frac{6300}{1.10^{-3}}} = \underline{400Hz}$$

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$$P_2 = \frac{|f_{acc}|^2}{8D_s} = \frac{(1 \cdot 10^{-3} \cdot 1 \cdot 10^{-4} \cdot (2\pi \cdot 400)^2)^2}{8 \cdot 0,17} = \underline{464\text{mW}}$$

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$$v_{max} = N R_L \omega_0 |w| = 0,012 \cdot 1180 \cdot 2\pi \cdot 400 \cdot 738 \cdot 10^{-6} = \underline{26\text{V}}$$

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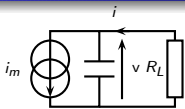
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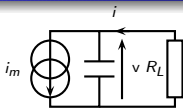
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- $Z_{Cb} = \frac{1}{C_b \omega} = \frac{1}{2\pi \cdot 400 \cdot 300 \cdot 10^{-9}} = \underline{1990\Omega} \approx R_L, \rightarrow$ **Calculations are NOT valid!**

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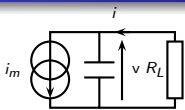


We can show:

$$\underline{v} = -\frac{R_L}{1+j\omega R_L C_b} \underline{i}_m = -j\omega N R_L \frac{1-j\omega R_L C_b}{1+(R_L C_b \omega)^2} \underline{w}_m$$

and we write: $\underline{v} = -j\omega N r_{Leq} \underline{w} - N k_{eq} \underline{w}$

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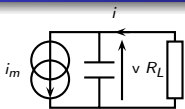
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2 Cases to consider, since ω should be ω_0 :

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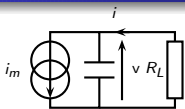
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2 Cases to consider, since ω should be ω_0 :

$$\omega_0 \ll \frac{1}{R_L C_b}$$

For a real Generator, $D_s \neq 0$ and $v \neq -R_L i_m$

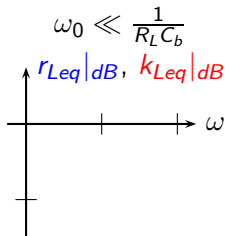


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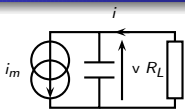
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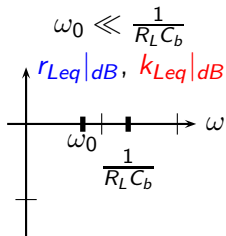


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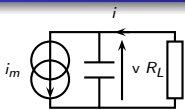
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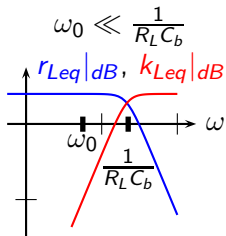


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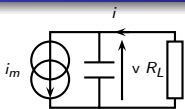
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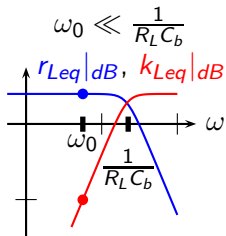


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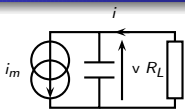
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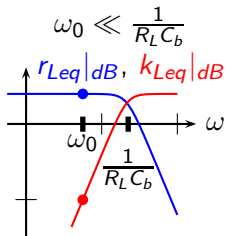


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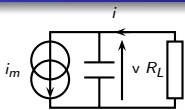
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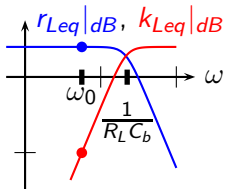
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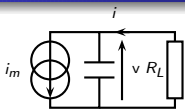
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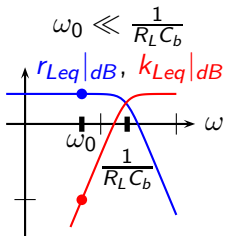


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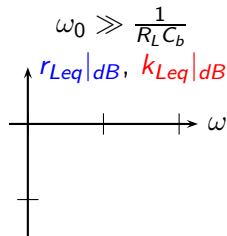
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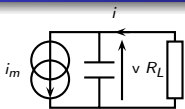
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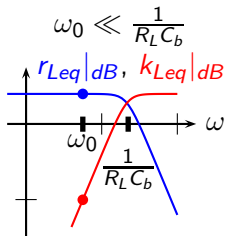


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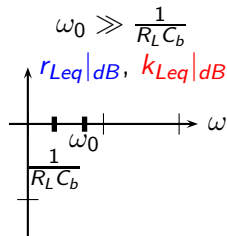
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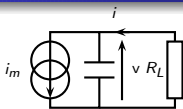
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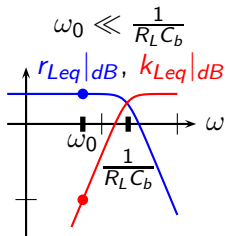


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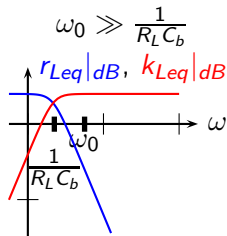
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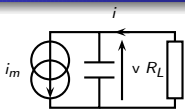
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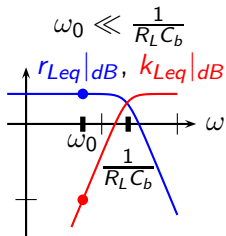


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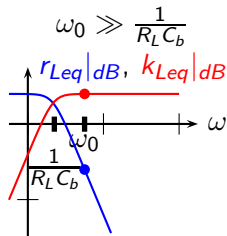
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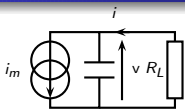
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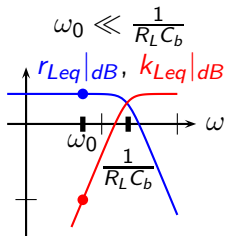


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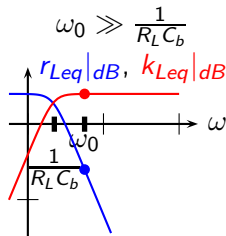
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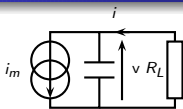


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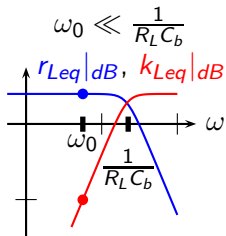


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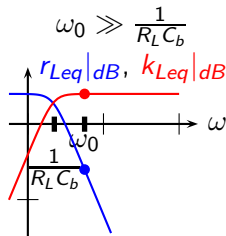
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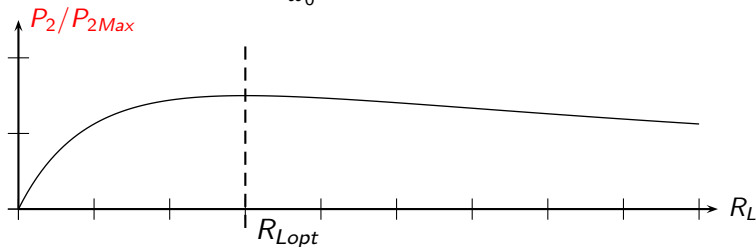
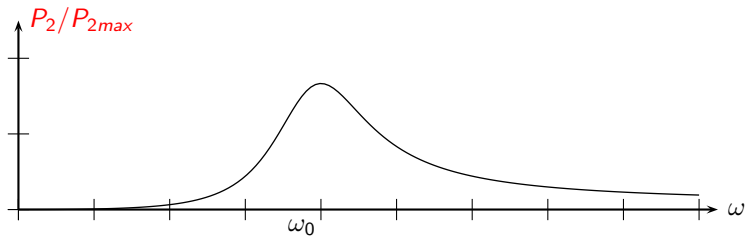


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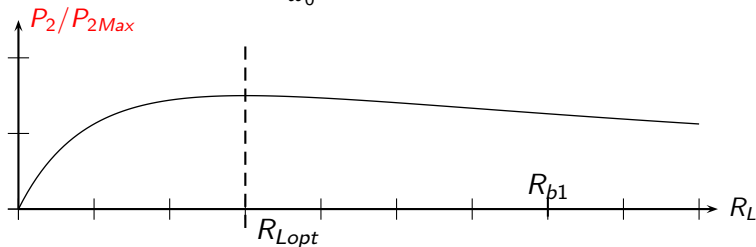
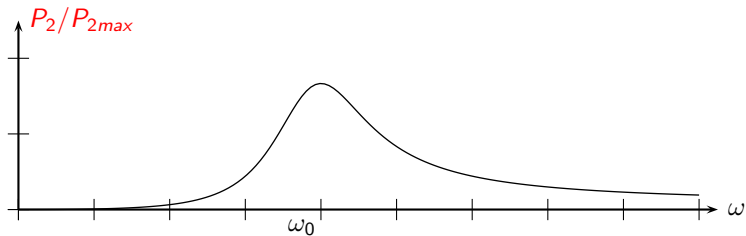
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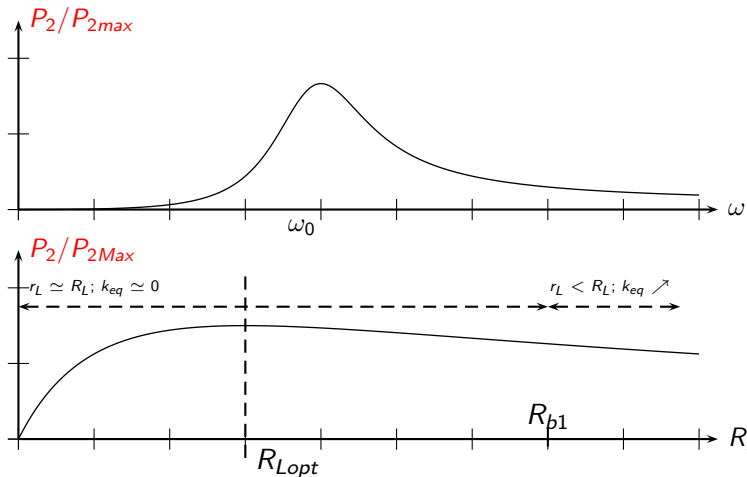
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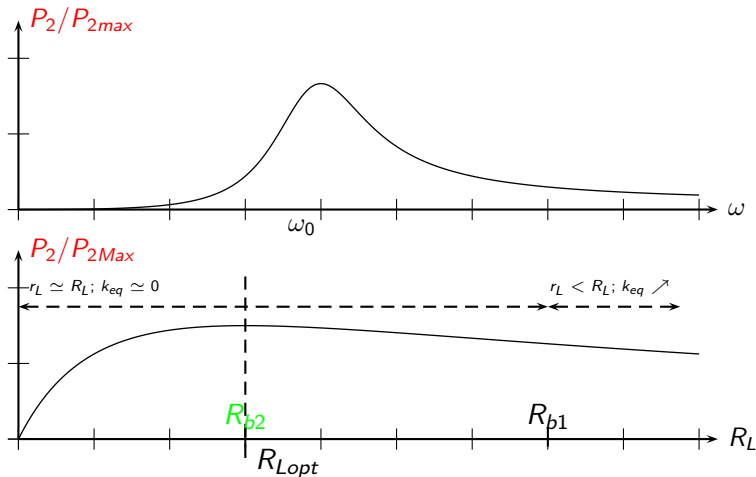
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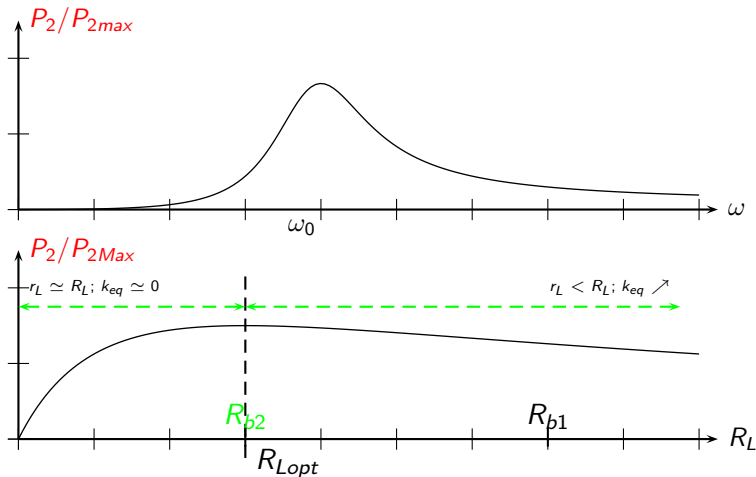
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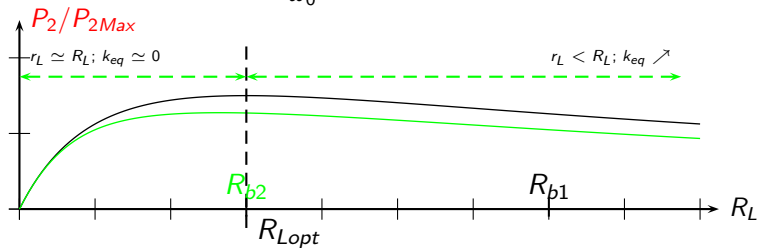
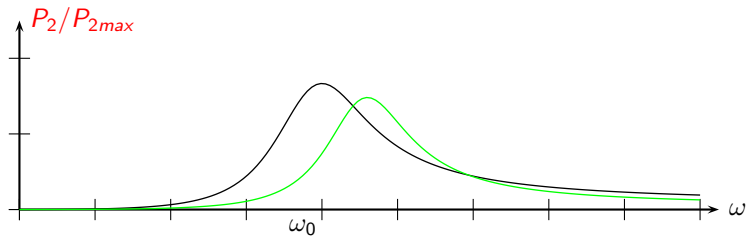
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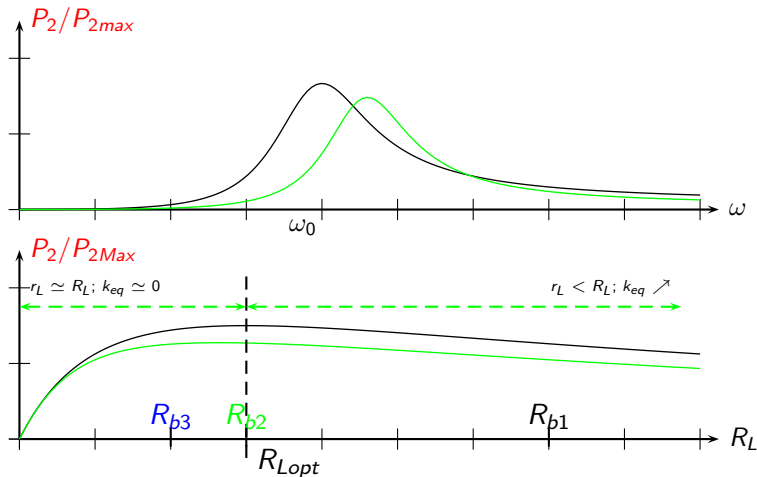
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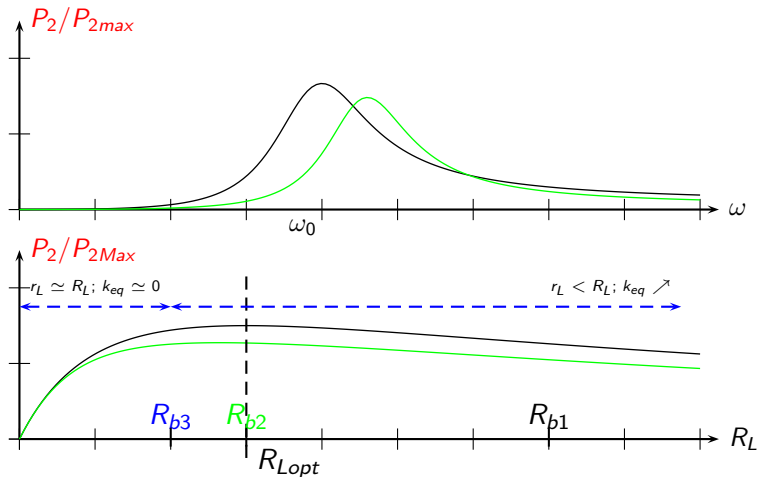
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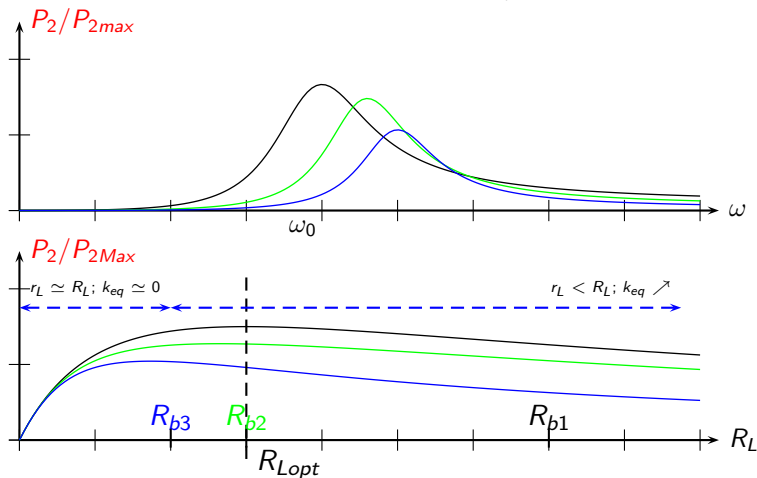
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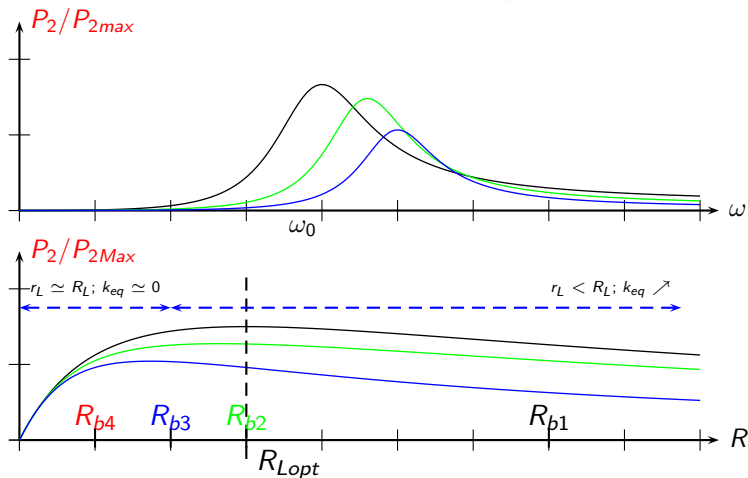
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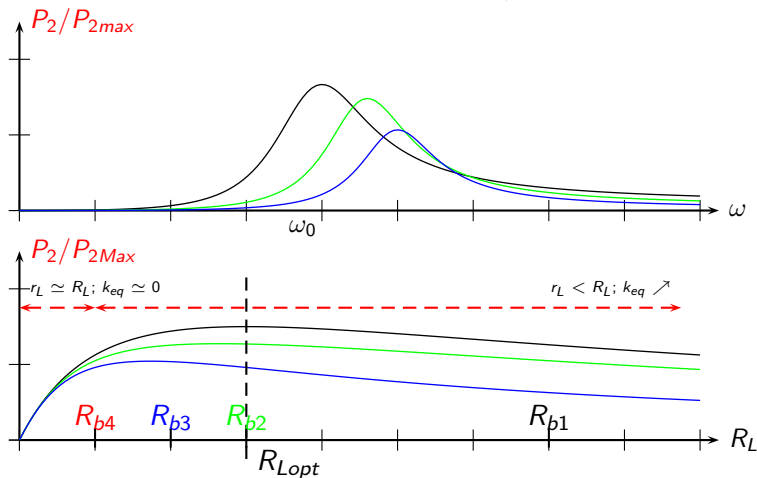
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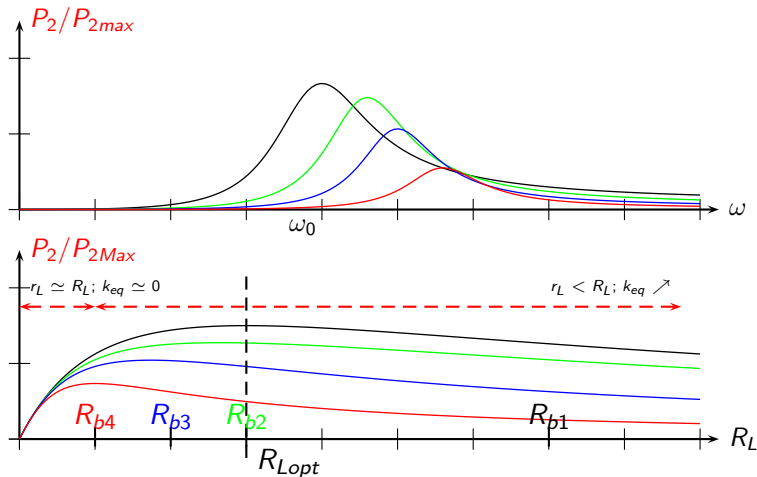
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Partial Conclusion

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If $R_{opt} > R_b$

The power is not well extracted because $P_2 < P_{2max}$. This happens for high damped mechanisms.

A simple resistor cannot recover the power optimally (and this is due to C_b).

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

resonance is shifted

- $r_{Leq} = \frac{R_L}{1+(R_L C_b \omega)^2} \simeq \frac{R_L}{(R_L C_b \omega)^2}$

- $k_{eq} = R_L \frac{\omega^2 R_L C_b}{1+(R_L C_b \omega)^2} \simeq \frac{1}{C_b}$

$((K_s + N^2 k_{eq}) - M\omega^2) + j\omega(D_s + N^2 r_{Leq})\underline{w} = \underline{f}_{acc}$ leads to:

$$\omega'_0 = \sqrt{\frac{K_s + N^2 k_{eq}}{M}} = \sqrt{\frac{K_s + \frac{N^2}{C_b}}{M}}$$

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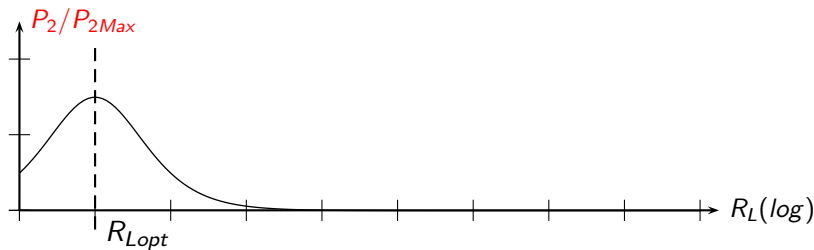
Another optimal resistor

The optimal power is harvested is $N^2 r_{Leq} = D_s$, leading to:

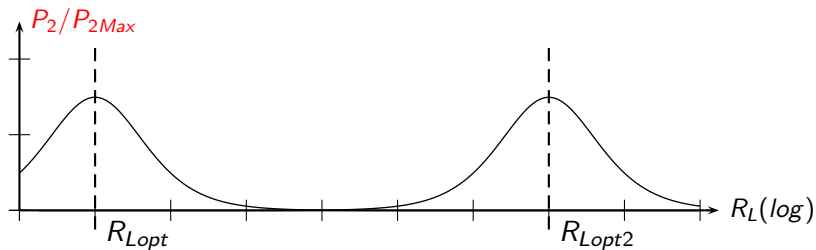
$$\frac{N^2 R_L}{(R_L C_b \omega'_0)^2} = \frac{N^2}{R_L C_b^2 \frac{K_s + \frac{N^2}{C_b}}{M}} = D_s$$

$$R_{Lopt2} = \frac{N^2}{D_s} \frac{1}{C_b^2 \frac{K_s}{M} (1 + \frac{N^2}{K_s C_b})} = \frac{R_b^2}{R_{Lopt}} \frac{1}{1 + \frac{N^2}{K_s C_b}}$$

What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?

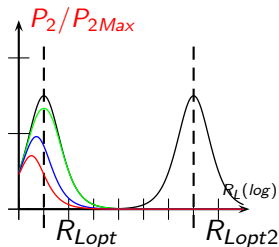


What happens if $R_{opt} < R_b$ and $R_L \gg R_b$?



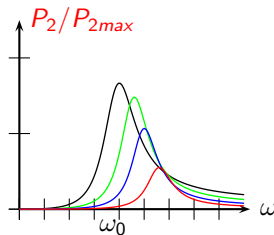
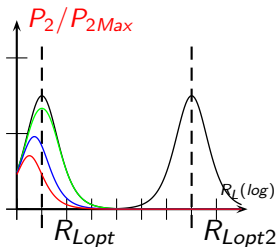
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- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,



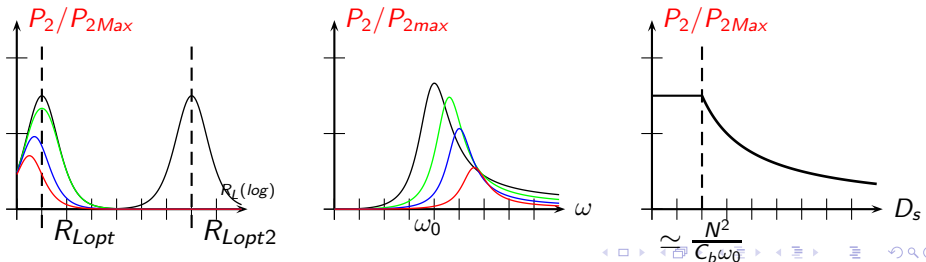
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- For the realistic case, there is one or two resistive loads which allow to extract the maximum of power,
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- we always want to harvest the maximum of power!

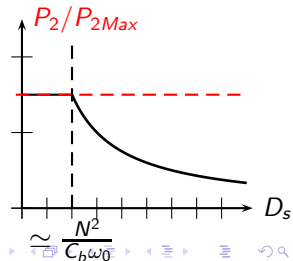
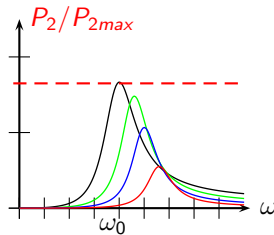
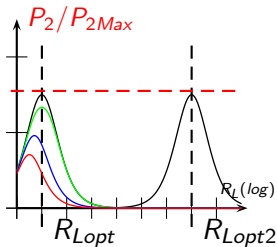
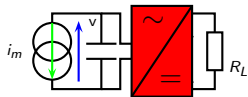


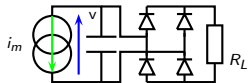
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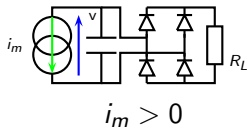
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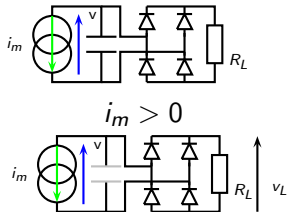
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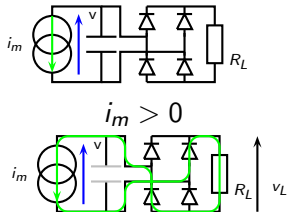
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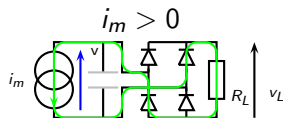
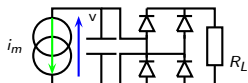
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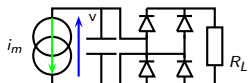
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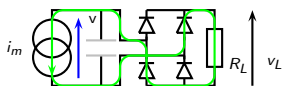
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$$i_m > 0$$

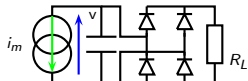


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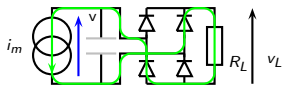
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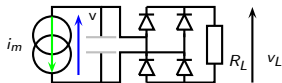
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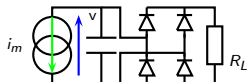
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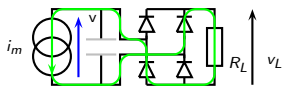
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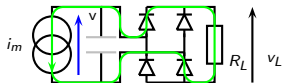
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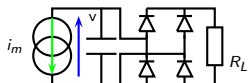
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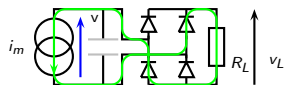
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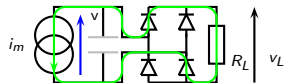
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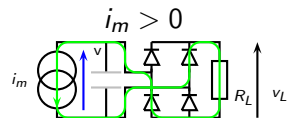
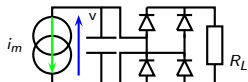


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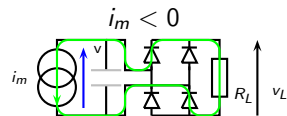
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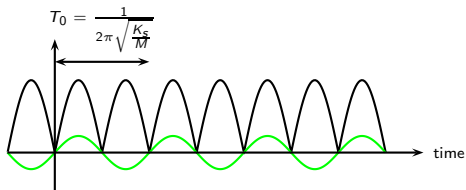
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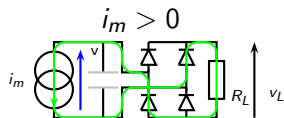
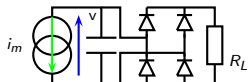
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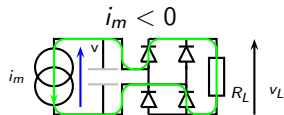
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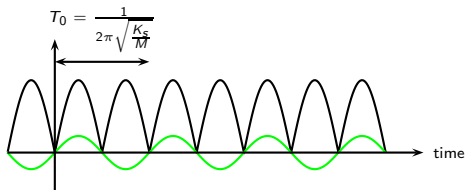
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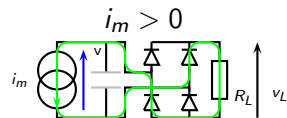
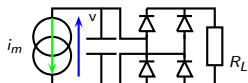


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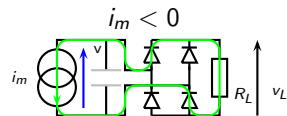
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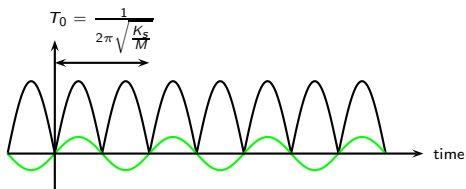
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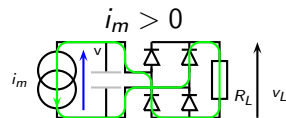
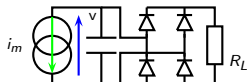
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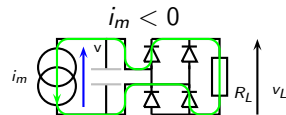
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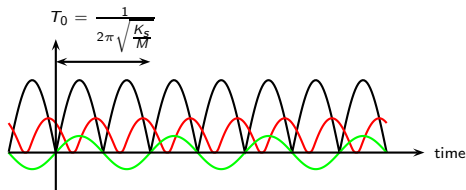
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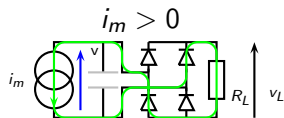
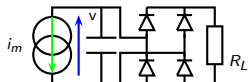
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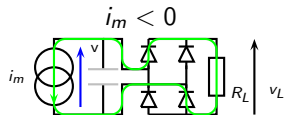


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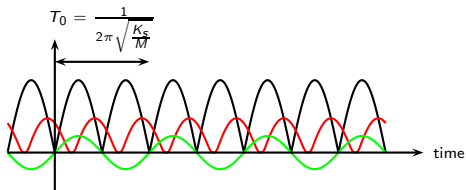
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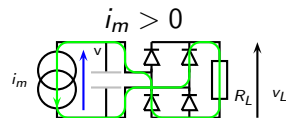
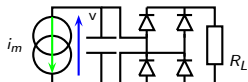


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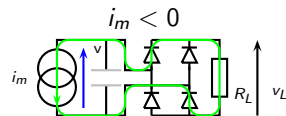


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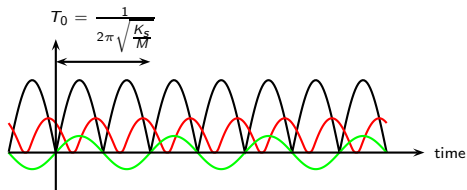
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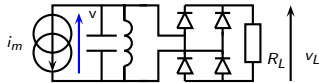
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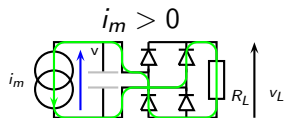
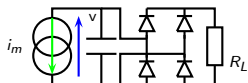
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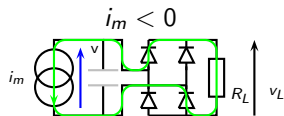
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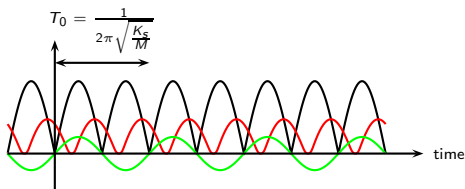
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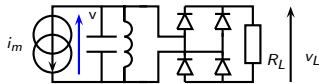
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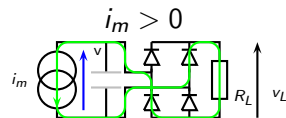
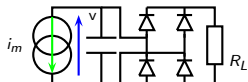


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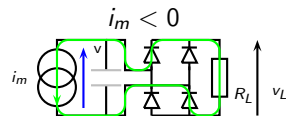


This is a bad solution because it works only for ω_0 (what if the frequency shifts?), and the Inductor is large (because ω_0 usually is small)

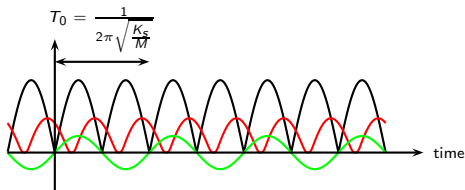
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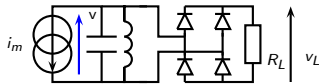
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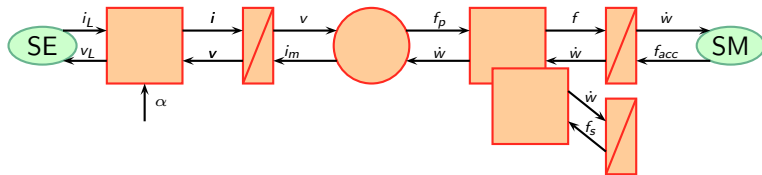
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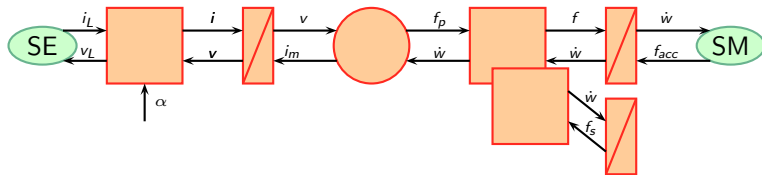
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→ Non linear Techniques

Why Synchronized

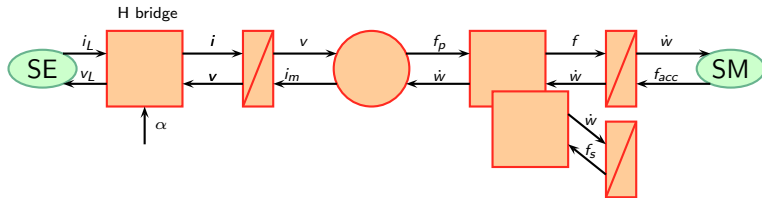
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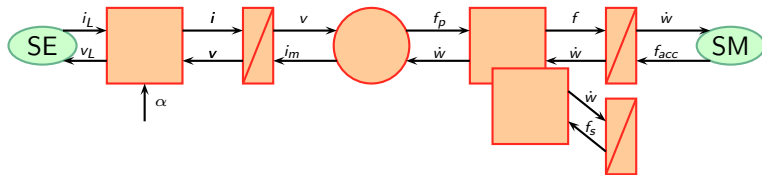
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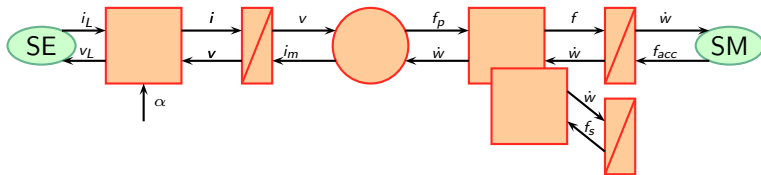


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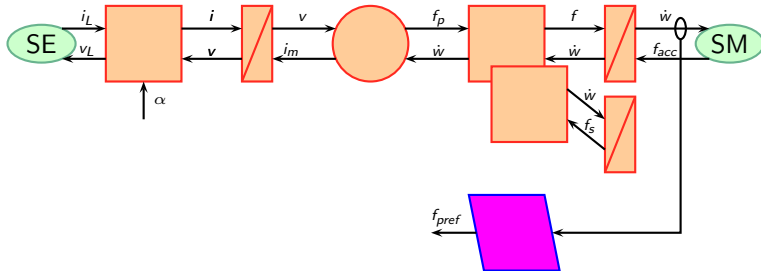
Strategy: $v = -R_{opt}i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2}N\dot{w}$ or, $f_p = -D_s\dot{w}$.

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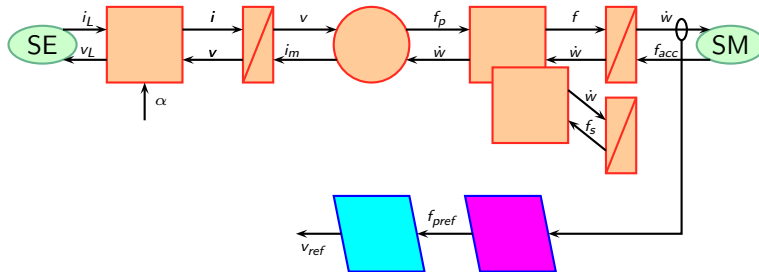
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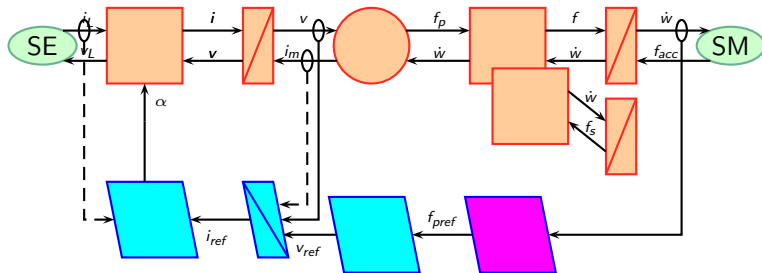
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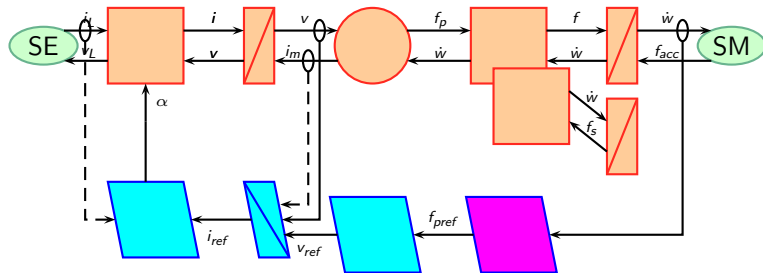
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Strategy: $v = -R_{opt} i_m$ leads to $\frac{f_p}{N} = -\frac{D_s}{N^2} N \dot{w}$ or, $f_p = -D_s \dot{w}$.
 This shows that v should be controlled. SSHI does this with efficiency.

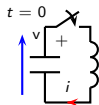
How it works

LC oscillations



How it works

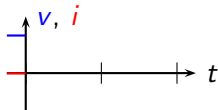
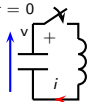
LC oscillations



How it works

LC oscillations

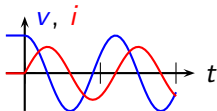
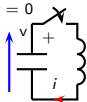
$t = 0$



How it works

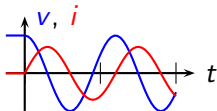
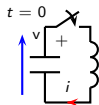
LC oscillations

$t = 0$



How it works

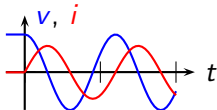
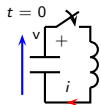
LC oscillations



Switched inductor

How it works

LC oscillations

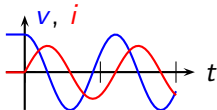
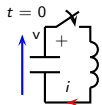


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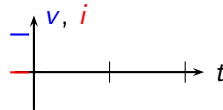
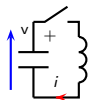


How it works

LC oscillations

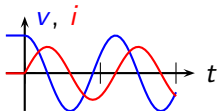
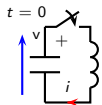


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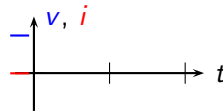


How it works

LC oscillations

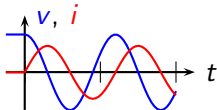
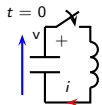


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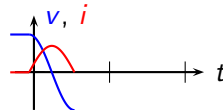


How it works

LC oscillations

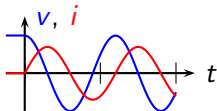
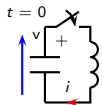


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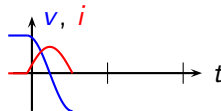
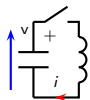


How it works

LC oscillations

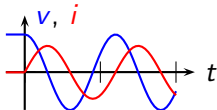
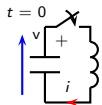


Switched inductor

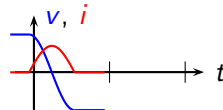
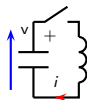


How it works

LC oscillations

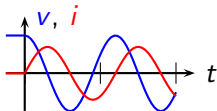
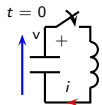


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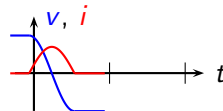


How it works

LC oscillations

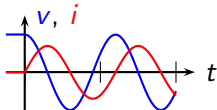
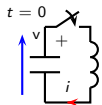


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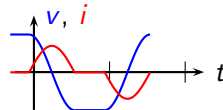


How it works

LC oscillations

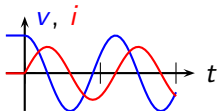
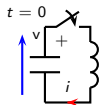


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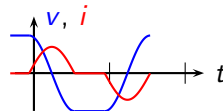
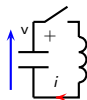


How it works

LC oscillations

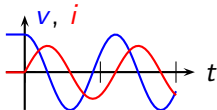
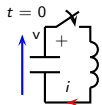


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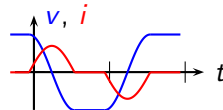
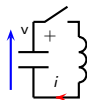


How it works

LC oscillations

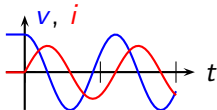
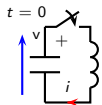


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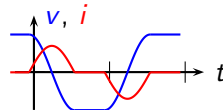
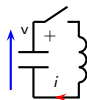


How it works

LC oscillations



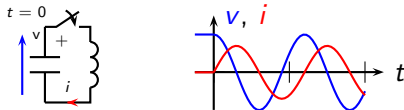
Switched inductor



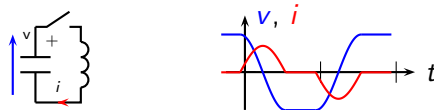
SSHI

How it works

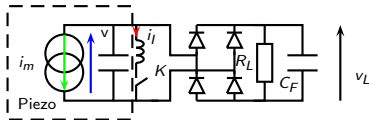
LC oscillations



Switched inductor

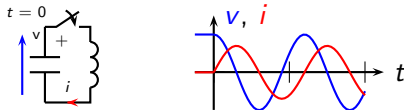


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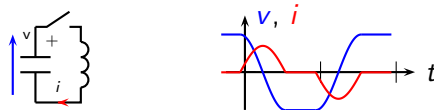


How it works

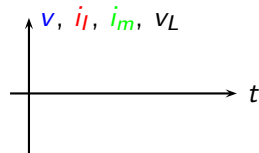
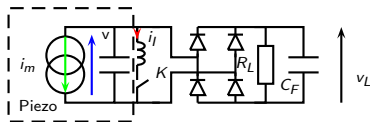
LC oscillations



Switched inductor

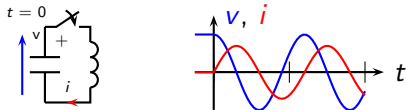


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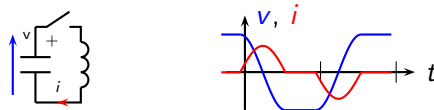


How it works

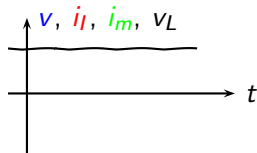
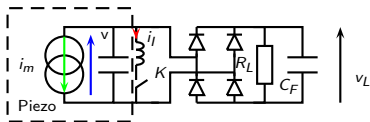
LC oscillations



Switched inductor

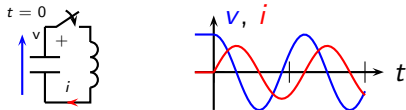


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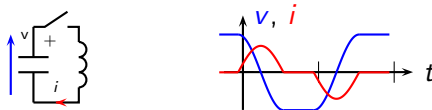


How it works

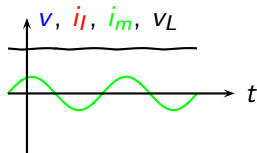
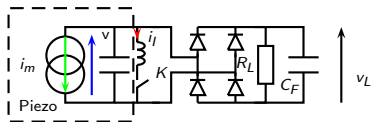
LC oscillations



Switched inductor

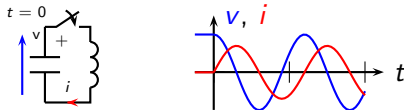


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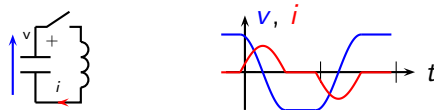


How it works

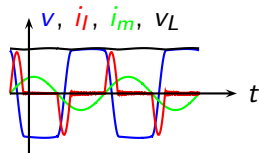
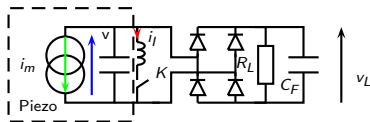
LC oscillations



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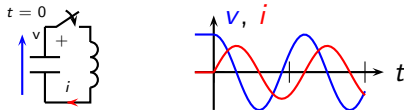


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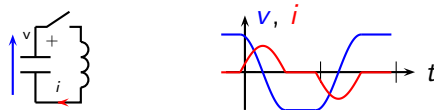


How it works

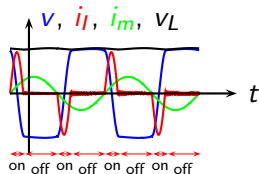
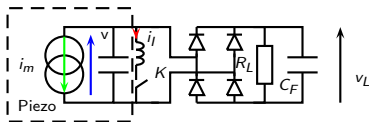
LC oscillations



Switched inductor



SSHI

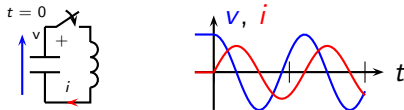


Switching is synchronized on \dot{w} , or

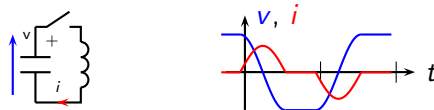
$$i_m = N\dot{w}.$$

How it works

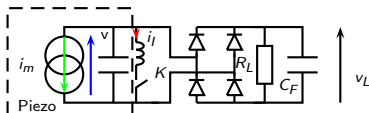
LC oscillations



Switched inductor

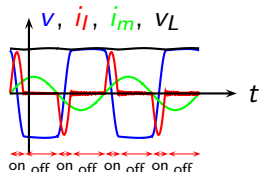


SSHI



Operating point: calculate v_L from i_m

1st Harmonic assumption: $P = \frac{1}{2} \frac{V_L^2}{R_L} \simeq \frac{1}{2} \frac{4V_L I_m}{\pi}$, $V_L = \frac{4}{\pi} R_L I_m$

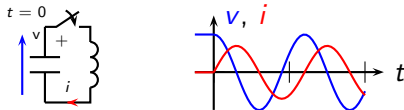


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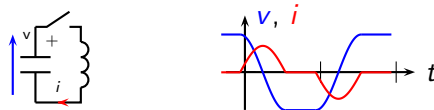
$$i_m = N\dot{w}$$

How it works

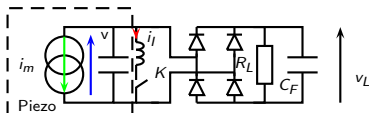
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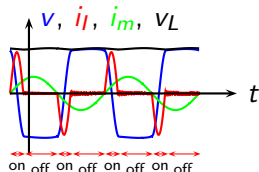
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SSHI controls v and synchronises it, but doesn't impose $f_p = -D_s \dot{w}$.



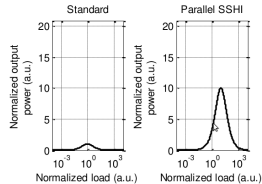
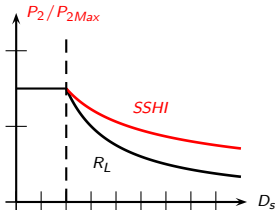
Switching is synchronized on \dot{w} , or

$$i_m = N \dot{w}$$

Conclusion

Performances

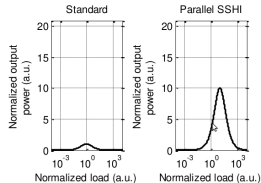
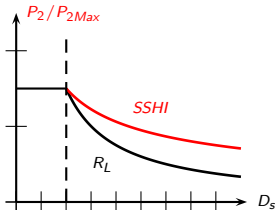
- SSHI can extract energy more efficiently than a resistor when damping is important,



Conclusion

Performances

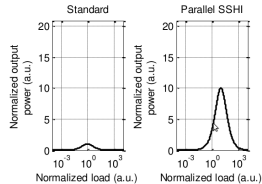
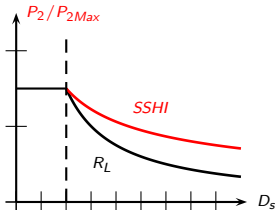
- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,



Conclusion

Performances

- SSHI can extract energy more efficiently than a resistor when damping is important,
- But Power extraction still depends on the load,
- Needs to measure bender's deflection $w(t)$.



General conclusion



In this presentation, applications of Energy Harvesting were shown. The modelling of a piezoelectric generator has shown that the power source needs an adaptation:

- in frequency,
- in load.

to maximize the harvested power.

The key energy management rules were presented through the analysis of the EMR of the system. A typical power electronic circuit was also presented, but the bibliography shows a lot of example.

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End of the presentation

Questions?