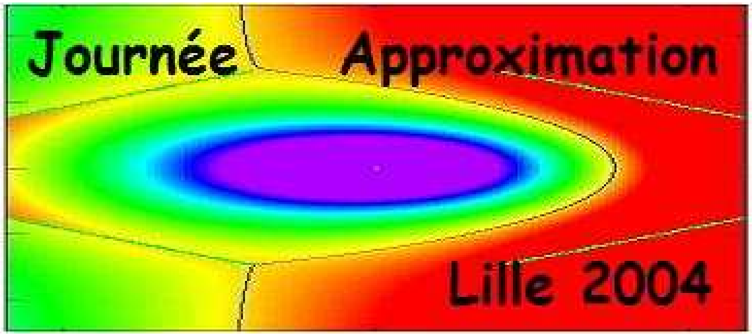


$2^{\text{ème}}$ Journée Approximation
Jeudi 25 mars 2004, 9h00 - 18h00
Salle de Réunion, Bâtiment M2

Equipe ANO-EDP
Laboratoire Paul Painlevé
UMR 8524
Université de Lille 1, France

SCOPE

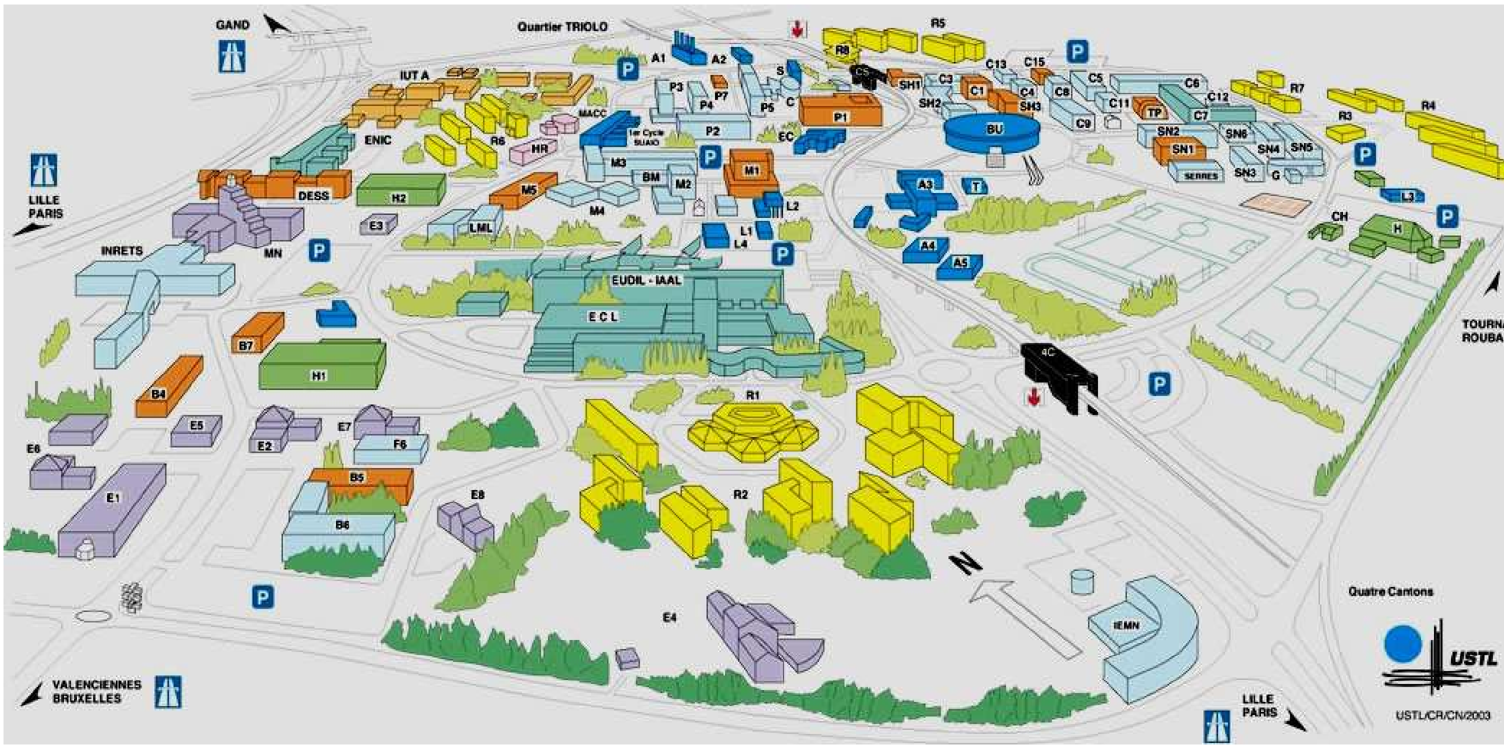
The aim of this meeting, the second one after a similar meeting in 2000, is to bring together people interested in Approximation Theory. New and recent work will be presented, together with its interaction with complex analysis, number theory and functional analysis

LOCATION OF THE WORKSHOP

The workshop takes place at the Université des Sciences et Technologies de Lille 1, Salle de Réunion, Bâtiment M2 (first floor), Cité Scientifique, Villeneuve d'Ascq. This lecture hall is located in the main building of the department of mathematics (see the map below Figure 1). For more details of how to join us, please see our web site

`http://ano.univ-lille1.fr/journee_approximation/ja2.html`

Organizers
Bernhard Beckermann, Claude Brezinski, Ana C. Matos, Jeannette Van Iseghem, Franck Wielonsky,



The campus

Social events

- Wednesday 24 March at 8h30 p.m. we have booked to have dinner in the restaurant “Aux Moules” in downtown Lille (34 rue de Béthune)
- Thursday 25 March at 8h30 p.m. we made a reservation to have dinner in the restaurant “Le Meunier” (15 Rue de Tournai, close to the railway station Lille Flandres)
- and for those staying friday morning a cultural program: a visit to the Rubens Exhibition in the Palais des Beaux Arts de Lille.

PROGRAM

9h00 - 9h05	Opening session
9h05 - 9h35	Tanguy Rivoal (U Caen) : On the denominator conjecture
9h40 - 10h10	Walter Van Assche (KU Leuven): Differential and difference equations for multiple orthogonal polynomials
10h15- 10h45	Pascale Vitse(U. Besançon): Free interpolation by polynomials of a given degree
10h50 - 11h20	Coffee break
11h20 - 12h05	Alexander Aptekarev (U. Moscou): Trajectories of quadractic differentials on algebraic Riemann surfaces and Hermite-Padé approximants
12h10 - 12h40	Reinhold Kustner (UST Lille): On the zero distribution of certzin non-Hermitian orthogonal polynomials
12h45 - 14h15	Lunch (Restaurant universitaire Charles Barrois)
14h15 - 15h00	Laurent Baratchart(INRIA Nice): Singular vectors of Hankel operators and normality
15h05 - 15h35	Stefan Becuwe (IU Anvers): Fast and validated computation of multivariate rational approxiamnts with applications
15h50 - 16h20	Coffee Break
16h25 - 17h10	Herbert Stahl (TFH Berlin): Hermite-Padé Polynomials associated with the exponential function
17h15 - 17h45	Franck Wielonsky (UST Lille): Entropy of orthogonal polynomials in the Szego class

ABSTRACTS

• Tanguy Rivoal (Université de Caen):

Title: *On the denominator conjecture*
Abstract:
Sans cet exposé j'exposerai deux méthodes permettant de construire de très bonnes approximations rationnelles de la constante de Catalan $G = \sum_{k\geq 0}(-1)^k/(2k+1)^2$: l'une des méthodes utilise une série hypergéométrique très bien équilibrée, la seconde des approximants de Padé.

Numériquement, la première construction n'est pas optimale en un sens précis ("Conjecture des dénominateurs") : j'indiquerai comment, de façon surprenante, la deuxième construction permet de prouver cette conjecture.

Malheureusement, ces approximations rationnelles ne suffisent pas à montrer l'irrationalité de G, qui reste un problème ouvert.

• Walter Van Assche (University of Leuven):

Title: *Differential and difference equations for multiple orthogonal polynomials*
Abstract:
The classical orthogonal polynomials (Jacobi, Laguerre, Hermite) all satisfy a linear second order differential equation of Sturm-Liouville type. Other polynomials from the Askey table satisfy a second order difference or q-difference equation. We will show that several multiple orthogonal polynomials also satisfy a differential or difference equation, but of higher order.

• Pascale Vitse (Université de Besançon):

Title: *Free interpolation by polynomials of a given degree*
Abstract:
We give a polynomial version of the Carleson free interpolation theorem for $H^\infty(\mathbb{D})$, that is free interpolation by polynomials with controlled degree and norm (the uniform norm on the unit disc). More precisely, given a Carleson sequence $(\lambda_k)_{k\geq 1} \subset \mathbb{D}$, and given an integer n, what is the largest disc $D(0, r_n)$ such that one can interpolate at all points λ_k in this disc by a polynomial of degree at most n with a control of the norm independant of n. An answer is given by the following theorem:
Theorem *Let $(\lambda_k)_{k\geq 1} \subset \mathbb{D}$ be a Carleson sequence with constant δ, $\delta > 0$, and let $r < 1$. For every $(a_k)_{k\geq 1} \in l^\infty$, there exists a polynomial p of degree at most $\frac{M}{1-r}$ such that $\|p\|_\infty \leq C\|a\|_\infty$ and $p(\lambda_k) = a_k$ si $|\lambda_k| \leq r$, where M and C are some constants depending only on δ.*
The proof consists in an iterated approximation of a solution.

• Alexandre Aptekarev (University of Moscow):

Title: *Trajectories of quadratic differentials on algebraic Riemann surfaces and Hermite-Padé approxiamnts*

Abstract:

Rational Hermite-Padé approximants for a set of functions with separated branch points is considered. The poles of approximants and extra interpolation points are distributed along the extremal curves, which are the trajectories of quadratic differentials on algebraic Riemann surface. The Riemann surface depends on the disposition of the branch points of approximated functions. The most interesting phenomenon is the change of the genus of the Riemann surface when the branch points of the functions pass through certain critical dispositions. We make an attempt to describe these bifurcations in the special case of the two functions, each of them has two branch points.

• Reinhold Küstner (UST Lille):

Title: *On the zero distribution of certain non Hermitian orthogonal polynomials*

Abstract:

We are dealing with the zero distribution of non-Hermitian orthogonal polynomials which arise as the denominators of Padé / rational / meromorphic approximants of the Cauchy transform of a complex Borel measure having compact support K contained in the real line / in (-1,1). Under the assumption that the argument of the measure is the restriction of a function of bounded variation it is possibe to obtain geometric constraints for the zeros of the orthogonal polynomials which imply that all weak* limit measures of the corresponding zero counting (probability) measures have support included in K. With an additional hypothesis on the density of the measure it then follows that the zero counting measures converge weakly* to the logarithmic / hyperbolic equilibrium distribution of K.

• Laurent Baratchart (INRIA Nice):

Title: *Singular vectors of Hankel operators and normality*

Abstract:

On considère le problème de savoir quand les vecteurs singuliers normalisés de l'opérateur de Hankel $H^2 \to \bar{H}_0^2$ de symbole

$$f(z) = \int_K \frac{d\nu(\xi)}{z-\xi},$$

où K est un compact du disque unité ouvert, ont des facteur extérieurs qui forment une famille normale dans le complémentaire de K. Ceci est une question-clef pour déterminer le comportement asymptotique des pôles des meilleurs approximants méromorphes de f sur le cercle via la téorie AAK, et c'est aussi un témoin important de la nature des multiplicités. Par exemple, si la normalité de la famille ci-dessu a lieu, seul un nombre fini de pôles peut rester dans un compact disjoint de K, et les multiplicités sont bornées.

Dans le cas où K est une géodésique hyperbolique, le résultat sous hypothèse que l'argument de ν est à variation bornée est obtenu dans les thèses de F. Mandrea et R. Kustner.

On examinera le cas d'un contour symétrique général pour le potentiel de Green, et on émettra quelques hypothèses sur le cas de compact K plus généraux.

• Stefan Becuwe (University of Antwerpen):

Title: *Fast and validated computation of multivariate rational approxiamnts with applications*
Abstract:

Fast and validated computation of multivariate approximants with applications

In this talk we discuss two applications that each need another type of rational approximant and in the same time we sketch some of the research carried out in Antwerp.

In the first application we try to model microwave circuits. A bivariate model is constructed. The model is being updated in a specific way till a given criterion is met. This new way of updating achieves much better results than those known before. The underlying technique is multivariate rational interpolation.

The second application is shape reconstruction. The relationship between the Radon transform, the Markov transform and the 2-dimensional Stieltjes transform, together with the Pade slice theorem, lets us reconstruct objects without using interpolation techniques, as was done in the past. To actually compute the reconstruction, we need multivariate homogeneous Pade approximants.

We describe a method, based on GKO, to compute both multivariate approximants with the same tools. A verification step is added to obtain a validated result.

• Herbert Stahl (TFH Berlin):

Title: *Asymptotics of Hermite-Padé Polynomials*
Abstract:

Hermite-Padé polynomials and approximants are in a very natural way generalizations of Padé approximants and continued fractions. Historically, they are, perhaps, most famous for their role in Hermite's proof of the transcendency of the number e, and for a long time they have only been a topic for specialists in analytic number theory.

Within the last 15 years, however, there has been a considerable up-swing of interest in this topic in complex and constructive approximation theory, where the field is typically connected with questions like multiple orthogonality, higher order recurrence relations, or the approximation of functions with branch points. But despite of much progress, many of the basic questions about convergence and asymptotics are still open.

The talk will be based on recent research about quadratic Hermite-Padé polynomials associated with the exponential function. After a somewhat broader introduction, new results are presented, and some time will be spent with the main tools that have made progress possible. A prominent place is taken here by a concrete compact Riemann surfaces of an algebraic function of third degree. Looking ahead to investigations of problems beyond the quadratic case will demand the analysis of more complex compact Riemann surfaces.

• Frank Wielonsky (UST Lille):

Title: *Entropy of orthogonal polynomials in the Szegő class*

Abstract:

We describe an asymptotic upper bound as $n \to \infty$ for the entropy integral, that appears e.g. in the study of the wave function of some important systems in quantum mechanics,

$$E_n(w) = -\int p_n^2(x) \log(p_n^2(x)) w(x) dx,$$

where p_n is the nth degree orthonormal polynomial with respect to a weight $w(x)$ on $[-1, 1]$ which belongs to the Szegő class. The entropy integral may be decomposed in a natural way as a sum of two functionals which are also studied, as well as some universal behavior (i.e. independent of the weight) of related quantities. When the weight belongs to the Bernstein class, the asymptotic properties can be completely described.

This is a joint work with B. Beckermann, A. Martinez–Finkelshtein, and E.A. Rakhmanov.