AMF2018 Proceedings

May 28, 2018

Conference Program

MATRIX AND TENSOR FUNCTIONS IN CONFLICT WITH APPROXIMATION

Lars Grasedyck

Institute for Geometry and Practical Mathematics
RWTH Aachen University, Germany
lgr@igpm.rwth-aachen.de

ABSTRACT

In this talk we will discuss several aspects and problems concerning the approoximate computation of (matrix) functions in structured (low rank) formats. First, we introduce structured low rank formats for the data-sparse representation of vectors, matrices and tensors. These are low rank factorizations for matrices, hierarchical low rank formats for tensors and tensorized low rank formats for vectors. Subsequently, we aim at computing approximations to functions of these, e.g. the rather simple element-wise (Hadamard) product. These serve as simple model problems for the general approximate arithmetic in structured formats as it arises in many fields, e.g. the solution of parameter dependent PDEs or quantification of uncertainties in real world applications. As it turns out the conflict with truncation or in other words the numerical instability plays a crucial role — as it is well known. But also the quite delicate choice of norms (although being equivalent) is even more pronounced for higher-dimensional or large scale problems. We will illustrate this by the straight-forward computation of the (absolute) largest element of a matrix, tensor, vector, and observe a few unresolvable issues. Despite of these we consider heuristics that can either be efficient (but possibly unreliable) or reliable (but possibly inefficient). We conclude with open probems for discussion.

A RATIONAL QZ ALGORITHM

Raf Vandebril

Department of Computer Science
University of Leuven, Belgium
`Raf.Vandebril@kuleuven.be`

ABSTRACT

We propose a rational QZ method for the solution of the dense, unsymmetric generalized eigenvalue problem. This is a generalization of the classical QZ method operating implicitly on a Hessenberg, Hessenberg pencil instead of on a Hessenberg, triangular pencil. Whereas the QZ method performs nested subspace iteration driven by a polynomial, the rational QZ method allows for nested subspace iteration driven by a rational function. This creates the additional freedom of selecting poles exhibiting interesting numerical properties.

The algorithm links to rational Krylov subspaces allowing us to prove essential uniqueness (implicit Q theorem) of the rational QZ iterates as well as convergence of the proposed method.

Numerical experiments are included to illustrate competitiveness in terms of speed and accuracy with the classical approach. Numerical experiments show competitiveness of the approach. Moreover, we illustrate that good pole selection can be used to deflate the original problem during the reduction phase and can lead to faster convergence.

These results have be obtained jointly with Daan Camps (University of Leuven) and Karl Meerbergen (University of Leuven).

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

THE TWO-PERIODIC AZTEC DIAMOND AND MATRIX VALUED ORTHOGONAL POLYNOMIALS

Arno Kuijlaars

Department of Mathematics
University of Leuven, Belgium
arno.kuijlaars@kuleuven.be

ABSTRACT

Uniform domino tilings of the Aztec diamond have the arctic circle phenomenon: near the corners the pattern is fixed and only one type of domino appears, while in the middle there is disorder and all types appear. The transition is sharp with fluctuations described by the Tracy-Widom distributions.

In the two-periodic Aztec diamond the dominos have a two-periodic weighting and this creates a new phase in the large size limit, where correlations decay at an exponential rate. In recent work with Maurice Duits (KTH Stockholm) we analyze this model with the help of matrix valued orthogonal polynomials. We obtain a remarkably simple double contour integral formula for the correlation kernel that we can analyze in the limit to recover the three phases of the model and the fluctuations near the transition curves.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

COMPRESSING VARIABLE-COEFFICIENT EXTERIOR HELMHOLTZ PROBLEMS VIA RKFIT

Stefan Güttel

School of Mathematics
The University of Manchester, United Kingdom
stefan@guettel.com

ABSTRACT

The efficient discretization of Helmholtz problems on unbounded domains is a challenging task, in particular, when the wave medium is nonhomogeneous. We present a new numerical approach for compressing finite difference discretizations of such problems, thereby giving rise to efficient perfectly matched layers for nonhomogeneous media. This approach is based on the solution of a matrix-valued nonlinear rational least squares problem using the RKFIT method. We show how the solution of this least squares problem can be converted into an accurate finite difference grid within a rational Krylov framework. This is joint work with Vladimir Druskin and Leonid Knizhnerman.

EXPERIMENTS WITH HERMITE-PADÉ APPROXIMANTS

Marco Fasondini

School of Mathematics, Statistics & Actuarial Science
University of Kent, United Kingdom
M.Fasondini@kent.ac.uk

ABSTRACT

We present the results of numerical experiments with multivalued algebraic Hermite-Padé approximants. We consider the approximation of a multivalued function on its Riemann surface, the estimation of its branch point locations and the approximation of a discontinuous function on an interval. In the latter experiment, our results suggest that the Hermite-Padé approximants circumvent the Gibbs phenomenon. This is joint work with André Weideman.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

A HIGH THROUGHPUT POLYNOMIAL AND RATIONAL FUNCTION APPROXIMATIONS EVALUATOR

Silviu-Loan Filip

IRISA
Campus de Beaulieu, France
`silviu.filip@inria.fr`

ABSTRACT

We present an automatic system to evaluate functions in hardware via polynomial or rational approximations. These approximations are evaluated using Ercegovacs iterative E-method for solving banded linear systems. The polynomial and rational function approximation coefficients are optimized such that they satisfy the constraints of the E-method. We present several examples of practical interest. In each case, the most resource-efficient approximation is used.

SOLVING THE LAPLACE PROBLEM WITH THE AAA ALGORITHM

Abinand Gopal

Mathematical Institute
University of Oxford, United Kingdom
Abinand.Gopal@maths.ox.ac.uk

ABSTRACT

The AAA Algorithm introduced by Nakatsukasa, Sete, and Trefethen is a robust and efficient way of computing rational approximations to functions in the complex plane. The core idea is the use of the rational barycentric representation with adaptive choice of support points and minimization of a linearized residual. In this poster, we illustrate how AAA can be leveraged to construct approximations to solutions of the Laplace problem. We are given a domain and real-valued boundary data on the boundary of the domain, and seek to construct a rational function whose real part closely matches the prescribed boundary data and is analytic in the domain. Our strategy for this is to first neglect the analyticity constraint, and use AAA to construct a rational function which matches the boundary data as closely as possible. We then proceed by iteratively reducing the coanalytic part of this function to zero while maintaining accurate matching of the real part of the approximation with the boundary data using a similar approach to earlier work by Hochman, Leviatan, and White.

ORTHOGONAL SYSTEMS WITH A SKEW-SYMMETRIC DIFFERENTIATION MATRIX

Marcus Webb

Department of Computer Science
KU Leuven, Belgium
marcus.webb@cs.kuleuven.be

ABSTRACT

For certain time-dependent PDEs, the norm of the solution as time progresses necessarily decays, or is preserved, e.g. the diffusion, Schrdinger, or nonlinear advection equations, a property due to the skew-hermitian nature of the differentiation operator. For a numerical solution of these PDEs, these properties of the underlying analytical solutions can be perfectly respected if the matrix representing differentiation in your discretisation is skew-hermitian too. In this talk, we characterise all systems of orthogonal functions in $L_2(\mathbb{R})$ such that the differentiation matrix for an expansion in these functions is real, skew-symmetric, tridiagonal and irreducible, accomplished by interesting links between orthogonal polynomials, the Fourier transform, and Paley-Wiener band-limited function spaces. This is joint work with Arieh Iserles (Cambridge), with a preprint available at http://www.damtp.cam.ac.uk/user/na/NA_papers/NA2018_02.pdf.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

SEQUENTIAL SAMPLING FOR KERNEL MATRIX APPROXIMATION AND ONLINE LEARNING

Michal Valko

SequeL
Inria Lille - France
michal.valko@inria.fr

ABSTRACT

Most kernel methods, such as kernel regression, kernel PCA, ICA, or second-order gradient methods, do not scale to large datasets, because of the costly construction and storing of the kernel matrix K_n. Prior work (Alaoui & Mahoney 2015, Musco et al 2016) showed that sampling points according to their ridge leverage scores (RLS) generates small dictionaries with strong spectral approximation guarantees for K_n. However, computing exact RLS requires constructing and storing the whole kernel matrix.

In this talk, we will talk about SQUEAK (AISTATS 2017), a new algorithm for kernel approximation based on RLS sampling that sequentially processes the dataset, storing a dictionary with a number of points that only depends on the effective dimension $d_{\text{eff}}(\gamma)$ of the dataset. Moreover, since all the RLS estimations are efficiently performed using only the small dictionary, SQUEAK never constructs the whole matrix K_n, runs in linear time $O(n * d_{\text{eff}}(\gamma)^3)$ w.r.t. n, and requires only a single pass over the dataset. A distributed version of SQUEAK runs in as little as $O(\log(n) * d_{\text{eff}}(\gamma)^3)$ time.

This is joint work with Daniele Calandriello and Alessandro Lazaric.

ON CONVERGENCE OF JACOBI-TYPE ALGORITHMS FOR SIMULTANEOUS ORTHOGONAL SYMMETRIC TENSOR DIAGONALIZATION

Konstantin Usevich

Centre de Recherche en Automatique de Nancy
UMR (mixed research unit) of CNRS and University of Lorraine - France
konstantin.usevich@univ-lorraine.fr

ABSTRACT

Symmetric tensors (or sets of symmetric matrices), in general, cannot be diagonalized (jointly diagonalized) by orthogonal transformations. In this work, we consider a family of Jacobi-type algorithms for simultaneous orthogonal diagonalization problem of symmetric tensors. For the Jacobi-based algorithm of [SIAM J. Matrix Anal. Appl., 2(34):651–672, 2013], we prove its global convergence for simultaneous orthogonal diagonalization of symmetric matrices and 3rd-order tensors. We also propose a new Jacobi-based algorithm in the general setting and prove its global convergence for sufficiently smooth functions.

This is joint work with Jianze Li and Pierre Comon (GIPSA-lab, CNRS and Univ. Grenoble Alpes). The preprint is available at https://arxiv.org/abs/1702.03750.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

NEWTON-LIKE VALIDATION METHOD FOR CHEBYSHEV APPROXIMATE SOLUTIONS OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS

Florent Bréhard

Laboratoire de l'Informatique du Parallélisme
ENS de Lyon - France
`florent.brehard@ens-lyon.fr`

ABSTRACT

A wide range of efficient numerical routines exist for solving function space problems (ODEs, PDEs, optimization, etc.) when no closed form is known for the solution. While most applications prioritize efficiency, some safety-critical tasks, as well as computer assisted mathematics, need rigorous guarantees on the computed result. For that, rigorous numerics aims at providing numerical approximations together with rigorous mathematical statements about them, without sacrificing (too much) efficiency and automation.
In the spirit of Newton-like validation methods (see for example [3]), we propose a fully automated algorithm which computes both a numerical approximate solution in Chebyshev basis and a rigorous uniform error bound for a restricted class of differential equations, namely Linear ODEs (LODEs). Functions are rigorously represented using Chebyshev models [2], which are a generalization of Taylor models [4] with better convergence properties. Broadly speaking, the algorithm works in two steps: (i) After applying an integral transform on the LODE, an infinite-dimensional linear almost-banded system is obtained. Its truncation at a given order N is solved with the fast algorithm of [5]. (ii) This solution is validated using a specific Newton-like fixed-point operator. This is obtained by approximating the integral operator with a finite-dimensional truncation, whose inverse Jacobian is in turn approximated by an almost-banded matrix, obtained with a modified version of the algorithm of [5]. As an example, we propose to validate a satellite trajectory arising in a space rendezvous problem (a more in-depth study is led in [1]). A C library implementation is freely available online (https://gforge.inria.fr/projects/tchebyapprox/).

References

[1] P. R. Arantes Gilz, F. Bréhard, and C. Gazzino. Validated Semi-Analytical Transition Matrix for Linearized Relative Spacecraft Dynamics via Chebyshev Polynomials. In 2018 Space Flight Mechanics Meeting, AIAA Science and Technology Forum and Exposition, page 24, 2018.
[2] N. Brisebarre and M. Joldes. Chebyshev interpolation polynomial-based tools for rigorous computing. In Proceedings of the 2010 International Symposium on Symbolic and Algebraic Computation, pages 147154. ACM, 2010.
[3] J.-P. Lessard and C. Reinhardt. Rigorous numerics for nonlinear differential equations using Chebyshev series. SIAM J. Numer. Anal., 52(1):122, 2014.
[4] K. Makino and M. Berz. Taylor models and other validated functional inclusion methods. International Journal of Pure and Applied Mathematics, 4(4):379456, 2003.
[5] S. Olver and A. Townsend. A fast and well-conditioned spectral method. SIAM Review, 55(3):462 489, 2013.

LARGE Z ASYMPTOTICS FOR SPECIAL FUNCTION SOLUTIONS OF PAINLEV II

Alfredo Deaño

School of Mathematics, Statistics & Acturial Science
University of Kent - United Kingdom
a.deano-cabrera@kent.ac.uk

ABSTRACT

The tau function corresponding to special function solutions of the Painlev II differential equation can be written as a Wronskian determinant involving classical Airy functions. Using Heines formula to rewrite this determinant as a multiple integral, defined over suitable contours in the complex plane, we obtain asymptotic approximations as z tends to infinity, using the classical method of steepest descent.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

NUMERICAL COMPUTATION WITH RATIONAL FUNCTIONS

Nick Trefethen

Oxford University Mathematical Institute
University of Oxford, United Kingdom
trefethen@maths.ox.ac.uk

ABSTRACT

New possibilities are opening up for computation with rational functions. In particular, with Beckermann, Filip, Gopal, and Nakatsukasa, I have been exploring extensions and applications of AAA approximation, in which the usual ratio of polynomials is replaced by a ratio of partial fractions with adaptively determined support points. The talk will present some of the highlights.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

RECONSTRUCTION OF NON-STATIONARY SIGNALS BY THE GENERALIZED PRONY METHOD

Gerlind Plonka-Hoch

Institute for Numerical and Applied Mathematics
University of Gottingen, Germany
plonka@math.uni-goettingen.de

ABSTRACT

In this talk, we reconsider the problem of parameter identification in short exponential sums which can be solved by the well-known Prony method. The exponential sum can be also interpreted as a sparse linear combination of eigenfunctions of the shift operator. This view led to a generalization of Pronys method in Peter & Plonka (2013), where we have shown that sparse expansions of eigenfunctions of linear operators can be reconstructed completely by using only a small number of suitable sample values. In this talk, we consider special classes of generalized shift operators and corresponding sets of eigenfunctions that admit a reconstruction of structured functions from function values. In particular, we can show that the reconstruction of expansions of shifted Gaussians, Gabor expansions with Gaussian window functions, Gaussians with different scaling as well as non-stationary signals with special monotone phase functions can be reconstructed by the generalized Prony method.

These results have be obtained jointly with Thomas Peter (University of Vienna), Kilian Stampfer, and Inge Keller (University of Gttingen).

SPARSE REPRESENTATIONS FROM MOMENTS

Bernard Mourrain

Inria
Sophia Antipolis Mditerrane, France
bernard.mourrain@inria.fr

ABSTRACT

Recovering a hidden structure from measurements, observations, evaluation, statistics etc. is a problem that is encountered in many domains such signal processing, medical imaging, geometric modeling. It has a long history going back to the work of G. de Prony on the decomposition of a function as a sum of exponential functions or the work of J.J. Sylvester on the decomposition of a binary form as a sum of powers of linear forms, or even more recently Berlekamp-Massey approach for decoding algebraic codes.

We consider the following related multivariate problem. Given a multi-index sequence of numbers, decompose its generating series as a sum of polynomial-exponential series.

We will show that polynomial-exponential series are naturally in correspondance with Artinian Gorenstein algebras. This leads to a generalization of Kronecker theorem for multivariate Hankel operators of finite rank, relating the rank of the operator with inverse systems of multiple points.

Using the properties of Hankel operators, we show how the symbol of the Hankel operator of finite rank can be decomposed as a polynomial-exponential series. By exploiting standard eigenvector methods for solving polynomial equations, we compute the frequencies and weights of the polynomial-exponential series, using moments of low degree of the series. A connection with orthogonal polynomial will be mentioned for the effective computation of range and kernels of these Hankel operators.

We will present the problem of low rank completion of Hankel matrices, and its characterization via a flat extension criteria. Its connection with the best low rank approximation of an Hankel operator will be discussed.

The approach will be illustrated in different multivariate decomposition problems: multivariate Prony decomposition method, reconstruction of weighted sums of Dirac measures, representation of multivariate polynomial-exponential functions from values, sparse interpolation of polylog functions, tensor decomposition.

THREE-FOLD SYMMETRIC POLYNOMIALS WITH AN ALGEBRAIC CLASSICAL BEHAVIOUR

Ana Filipa Loureiro

School of Mathematics, Statistics & Actuarial Sciences
University of Kent, United Kingdom
A.Loureiro@kent.ac.uk

ABSTRACT

I will discuss sequences of polynomials of a single variable that are orthogonal with respect to a vector of weights on a three-star of the complex plane. Such polynomial sequences satisfy a recurrence relation of finite (and fixed) order higher than 2. The main focus will be on polynomial sequences possessing a three-fold symmetry and whose multiple orthogonality is preserved under the action of the derivative operator. Among other things, I will also explain the asymptotic behaviour of the zeros of those polynomial sequences. This is a joint work with Walter Van Assche.

Proc. of the International conference on Approximation and Matrix Functions (AMF2018), Lille, France, Thursday May 31 and Friday June 1, 2018

STRUCTURED LOW-RANK APPROXIMATION APPROACH TO SUM-OF-EXPONENTIALS MODELING

Ivan Markovsky

Department ELEC
Vrije Universiteit, Brussel, Belgium
ivan.markovsky@vub.ac.be

ABSTRACT

The first part of the presentation is a tutorial introduction to sum-of-exponentials modeling from a numerical linear algebra point of view. The problem considered is related to the rational approximation, linear time-invariant system realization, and model reduction problems. Three classed of solution methods are reviewed: convex relaxations based on the nuclear norm heuristic, subspace methods, and local optimization methods. The class of subspace methods is related to Pronys method and its generalizations. The second part of the presentation shows generalizations of the structured low-rank approximation approach to modeling of systems with inputs, dealing with missing data, and an application to the problem of computing approximate common divisors of polynomials.

Index of Authors