Robust and Simple Log-Likelihood Approximation for Receiver Design

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***Abstract*—In impulsive noise, the inputs of the belief propagation decoder can be complex to compute or even impossible when the noise distribution is not known. We propose a simple approximation of the log-likelihood ratio that maps the channel output to the input of the error correcting decoder, for instance, LDPC decoders. This approximation is designed for additive impulsive noise channels, nevertheless, it is not computationally demanding and easy to be implemented. It requires the estimation of three parameters and we propose an efficient way to do it. Moreover, in terms of performance, our solution is barely discernible from the optimal receiver which is computationally prohibitive.**

***Index Terms*—Belief propagation, soft iterative decoding, impulsive interference, alpha-stable distributions.**

I. Introduction

With the denser deployment of wireless networks, the induced interference becomes the main system performance's limitation, due to the collection of undesired signals broadcasted by other transmitters. If the thermal noise caused by receiver equipment is well modeled by Gaussian distribution, it has been shown in many works that interference exhibits an impulsive behavior [1], [2], [3]. Many works have been proposed to tackle the modeling question, since the first works from Middleton [4] until the stochastic geometry solutions or symmetric alpha-Stable (SαS) distributions [2].

These works often lead to interference distributions that might be difficult to handle in receivers. Indeed the probability density function is sometimes expressed as an infinite series (Middleton) or has no closed-form expression (α-stable). Receivers frequently need the evaluation of the likelihood of the received sequence. However, in general, it cannot be simply evaluated, so that the design of an effective receiver scheme is complex. In literature, different approaches have been considered to overcome these issues. For instance, different demapping function or distance metrics like, p-norm [5], Hubber metric [6] or saturated likelihood [7]. If they improve performance, they remain difficult to implement, which induce much complexity.

The contribution of this paper is to propose a simple, fast and easy way to implement an approximation of the log likelihood ratio (LLR). In order to narrow the search space of such an approximation, we focus only on parametrized functions. Moreover, in order to avoid a noise-model decision step, we focus only on noise-model blind functions which can thus adapt between Gaussian or impulsive noise. More specifically, we propose a new parametrized function with three parameters to perform the LLR approximation which performs very close to the much more complex true LLR calculation. It also outperforms the most used LLR approximations, for instance, clipping demapper [8], soft limiter receivers, hole puncher and piecewise linear function [9] and the approximated LLR receiver [10]. This is illustrated using Low-Density Parity Check (LDPC) codes. They are associated with the Belief Propagation (BP) decoding algorithm, whose inputs are the LLR.

The rest of this paper is organized as follows: Section II presents the system model. Section III details our proposal. Section IV gives some simulation results using a regular LDPC code under impulsive noise and finally, Section V concludes the paper.

II. System Model and Background

A. System model

The system model comprises a transmitter, an additive impulsive noise channel and a receiver. The channel output Y, denotes the received message over a memoryless binary input symmetric-output channel (MBISO) that can be described by its conditional probability density function (pdf) $f_{Y|X}(y|x)$ with $f_{Y|X}(y|x = +1) = f_{Y|X}(-y|x = -1)$. Y is modeled by

$$Y = X + N \tag{1}$$

where X denotes the input message and N the additive noise.

The information source is first encoded using an LDPC encoder and is then mapped to a binary phase shift keying (BPSK) constellation. Throughout the paper, X is assumed to take its values in the alphabet $\{+1, -1\}$ with equal probability. N represents the interference that is assumed to be independent of X. In the remaining of the paper, N is modeled by an additive independent samples symmetric α-stable noise (AISαSN). In various environment types, the heavy tail property of the SαS has been shown coincide with the impulsive nature of network interference [11], [12], [2], [13]. At the receiver, different demappers are employed to mitigate the impulses in the received signal and generate LLRs used by the soft decision decoder.

The α-stable distribution can be seen as a generalization of the Gaussian distribution that accommodates for impulsive characteristics. One strong motivation of α-stable distribution models is provided by the Generalized Central Limit Theorem [14]. Moreover, the α-stable distributions family is stable

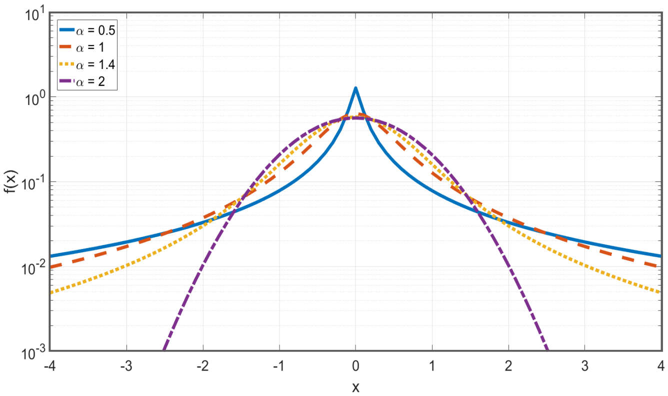

Fig. 1: Pdfs of SαS distributions for different values of α and $\gamma = 0.5$.

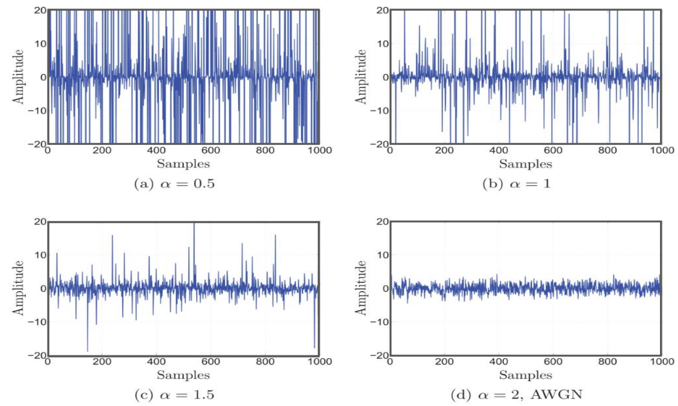

Fig. 2: Noise samples of SαS distributions $\gamma = 0.5$.

under linear combination for a given α, just like Gaussian variables which is the special case $\alpha = 2$. [15].

The characteristic function of a $S\alpha S$ random variable $\phi_{S\alpha}$ given as $\phi_{S_\alpha}(t) = \exp(-|\gamma t|^\alpha)$, depends on two parameters, the characteristic exponent α, where $(0 < \alpha \leq 2)$, and the dispersion γ, such that $\gamma > 0$. α sets the degree of impulsiveness of the distribution. The smaller the value of α, the heavier the tail of the pdf, which increases the likelihood of having impulses with large amplitudes and far from the central location. In wireless context, α is directly associated with the path loss exponent of the radio channel [16]. The dispersion γ is a scale parameter that measures the spread of the samples around the mean, similarly to the variance in the Gaussian case.

Several pdf of $S\alpha S$ distributions are plotted in Fig. 1 with a log-scale in the y-axis. Clearly, the smaller the value of the exponent α the heavier the tail becomes. The case where $\alpha = 2$ is the Gaussian case and the only to present an exponential decreasing tail which explains the very fast decrease of the probability of the extreme events.

Some realization of the noise distributed α-stable model for different values of α are given in Fig. 2 where the y-axis range is fixed to ease the comparison. It is clearly seen that this model is able to represent noises where large events are present and that decreasing α increases the importance of these large events.

B. Optimal and Suboptimal receivers

The LLR of the binary channel input X associated with the channel output Y under an additive noise channel is given by:

$$\mathrm{LLR}(y) = \log \frac{\Pr(Y = y|X = +1)}{\Pr(Y = y|X = -1)} = \log \frac{f(y-1)}{f(y+1)} \tag{2}$$

where $f(\cdot)$ is the pdf of the noise N. If the noise is modeled by a $S\alpha S$ variable, unfortunately, no closed-form expression of its pdf exists, consequently, the extraction of a simple metric based on the noise pdf in the decoding algorithm is not feasible.

The LLR can still be computed numerically, even if the pdf cannot be found in a closed-form expression, for instance, by numerical integration of the inverse Fourier transform of the characteristic function. The pdf of $S\alpha S$ random variable $x \sim S(\alpha, \gamma)$ is defined as:

$$f_\alpha(x;\gamma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-\gamma^\alpha |t|^\alpha) e^{-jtx}\, dt. \tag{3}$$

However, the integral in (3), induces a prohibitive computation and the evaluation of the LLR requires the knowledge of the noise parameters. Hence, a suboptimal receiver is then salutary to reduce the complexity.

In the literature, different implementations of suboptimal receivers have been proposed, as for example, Cauchy receiver [2] clipping demapper [8], piecewise linear solution [9] and the approximated LLR receiver [10]. The latter, which will be taken as the reference to compare with our proposed approximation gives the best performance overall approximations [17], [18].

A previous approximated LLR receiver which will be called thereafter L_{ab}, decomposes it into two parts: a linear part and an asymptotic part. The linear part is related to the noise dispersion γ and the asymptotic part reflects the heaviness of the tail,

$$L_{ab}(y) = \mathrm{sign}(y) \min\left(a|y|, \frac{b}{|y|}\right). \tag{4}$$

The main problem behind such approximations is the rough transition between the linear part and the asymptotic part, arising with a wrong LLR selection. Fig. 3 where $Pr(Y|X = +1)$ and $Pr(Y|X = -1)$ are plotted, indicates the problem raised by this transition, where roughly half of the received samples fall within this region. Consequently, to solve this problem we propose a new approximation based on three parameters expecting to have an approximation that fits the true LLR to a high degree without adding much complexity.

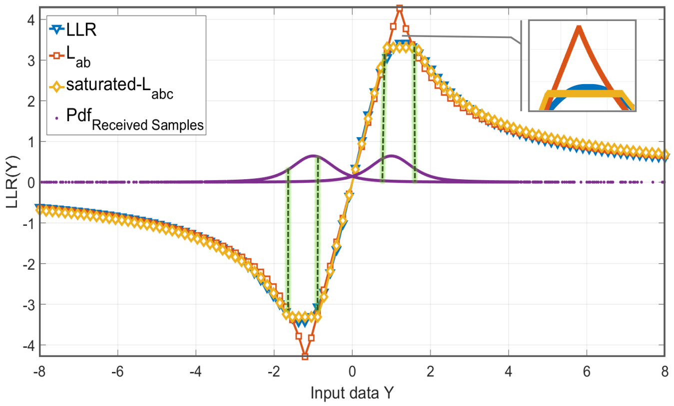

Fig. 3: Comparison of the LLR shapes under the effect of the estimated a and b parameters with $\gamma = 0.45$ and $\alpha = 1.4$, in the L_{ab} and L_{abc} approximations with the LLR obtained by numerical integration.

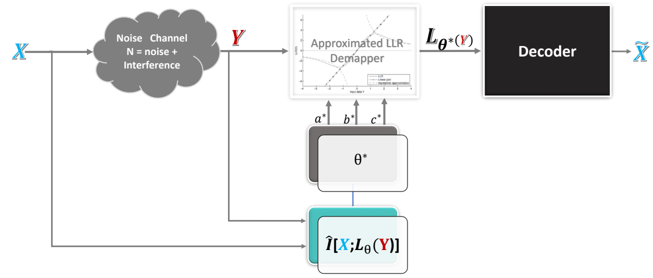

Fig. 4: Supervised LLR demapper.

III. Receiver Design

A. LLR approximation

In this Section, we will present our new proposed demapper L_{abc} that decomposes the LLR into three regions:

- First, when the channel output y is small the linear approximation of the LLR around zero is given by:

$$\begin{aligned}\text{LLR}(y) &= \log\frac{f(y-1)}{f(y+1)}\\ &= \log\frac{f(-1)+f'(-1)y+O(y^2)}{f(1)+f'(1)y+O(y^2)}\\ &= -2\frac{f'(1)}{f(1)}y+O(y^3)\approx ay.\end{aligned} \tag{5}$$

- Second, the asymptotic expansion of α-stable distribution given in [19, p. 16] provides a LLR approximation for large values of the channel output y,

$$\begin{aligned}\text{LLR}(y) &= \log\frac{\Pr(Y|X=+1)}{\Pr(Y|X=-1)} = \log\frac{f(\frac{y-1}{\gamma})}{f(\frac{y+1}{\gamma})}\\ &\approx \log\frac{(y-1)^{-(\alpha+1)}}{(y+1)^{-(\alpha+1)}}\\ &\approx 2\frac{\alpha+1}{y}\approx\frac{b}{y}.\end{aligned} \tag{6}$$

- The third part makes the transition between the linear and the asymptotic parts. As shown in Fig. 3, the true LLR behaves smoothly in this regions, whereas with the two aforementioned parts, the transition is rather sharp. In order to keep the simplicity and easy implementation of our demapper, we propose to introduce a new parameter c in order to introduce a saturation of the LLR.

Since almost half of the channel outputs fall around the transition region, improving the accuracy of the approximation in this area could significantly improve the performance. Moreover, the new parameter c avoids the cross point between the linear and asymptotic part, which allows a better fit of these two parts.

The three aforementioned points lead to the demapping function

$$\text{L}_{abc}(y) = \begin{cases} ay & \text{if } |y| < \sqrt{b/a},\\ c & \text{if } a|y| > c \text{ or } b/|y| > c,\\ b/y & \text{otherwise.}\end{cases} \tag{7}$$

B. Parameter estimation

To match the channel situation, the receiver must be tuned by the optimised parameters $\theta^* = (a, b, c)$.

In order to avoid direct estimation of the channel state to compute θ, we prefer to design the shape of the LLR directly from the channel output. The parameter optimization is performed using a mutual information maximization, since we have shown in [20], that this method achieves good performance compared to direct estimation and kernel density estimation. Supposing equally likely inputs, the capacity of MBISO channel is given by the mutual information between the input X and the channel output Y as $C = I(X, Y)$. For MBISO channel, the mutual information can be expressed as:

$$I_L(X, Y) = 1 - \mathbb{E}\left[\log_2(1 + e^{-XL(Y)})\right]. \tag{8}$$

where L denotes the LLR. By Replacing L with an approximated LLR L_θ, we get a lower bounded mutual information criterion which is given as:

$$\widehat{I}_{L_\theta}(X, Y) = 1 - \mathbb{E}\left[\log_2(1 + e^{-XL_\theta(Y)})\right]. \tag{9}$$

Authors in [21] proved that (9) reaches its maximum when the pdf of L_θ is equal to the pdf of true L.

Theoretically, finding the optimum LLR from the mutual information can be achieved by maximizing $\widehat{I}_{L_\theta}(X;Y)$. To narrow the search space of the best function, we look for the parametrized function L_θ as we just proposed, moreover, it gives us more flexibility to adapt for Gaussian and non-Gaussian channels. Thus, our objective is to fit the optimal L by finding θ that maximizes $\widehat{I}$ as:

$$\theta^* = \arg\max_\theta \widehat{I}_{L_\theta}(X;Y) \tag{10}$$

Since we do not make any assumption on the noise model, we cannot compute directly the expectation in (9); instead we propose to replace the expectation operator by an empirical

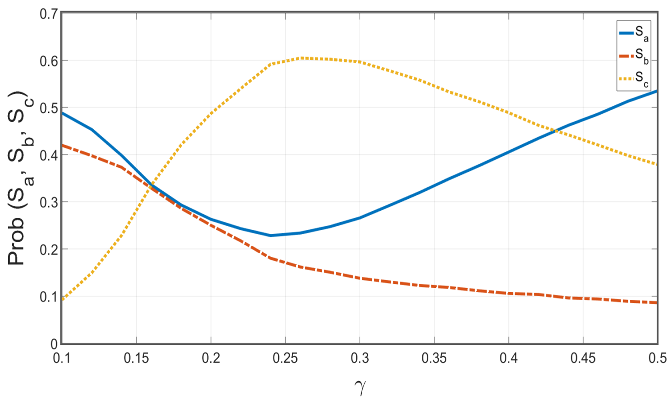

Fig. 5: Comparison of the mean evolution of the probability to fall within the a, b and c regions, as a function of the dispersion γ of a SαS noise with $\alpha = 1.8$ for the L$_{abc}$.

average. If the number of samples N is large enough, our approximation is known to be performant. Our optimization problem can thus be rewritten as

$$\begin{aligned} \theta^* &\approx \arg\max_\theta 1 - \frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + e^{-x_n L_\theta(y_n)}\right) \\ &\approx \arg\min_\theta \underbrace{\frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + e^{-x_n L_\theta(y_n)}\right)}_{f_{\text{opt}}(x_n;y_n)}, \end{aligned} \tag{11}$$

where x_n and y_n are samples that represent the input and output of the channel respectively.

The minimization of $f_{\text{opt}}(\cdot)$ will be tackled in our implementation via simplex method based algorithm [22]. Since one needs both an input sequence and the corresponding channel output in order to compute the mutual information, one can either perform a supervised optimization, where the input sequence is the training sequence [20] or a blind optimization, where an input sequence is build based on the channel output [23]. In this paper, we only consider the supervised optimization, so that the input sequence X is given as a learning sequence, but using the same arguments as in [23], it can be easily extended to the blind optimization. In order to study the degradation due to the estimation step, we will use a learning sequence of the size of the LDPC code.

Once the optimized parameter θ^* has been obtained, it will allow to obtain the estimated LLR that will feed the BP-algorithm as shown in Fig. 4.

Fig. 3 compares the LLR shapes obtained under supervised optimization with training sequence of 20000 sample, for L_{ab} ($a = 3.64, b = 5.19$) and L_{abc} ($a = 3.86, b = 5.5, c = 3.31$) to the true LLR obtained via numerical integration for a $S\alpha S$ noise of parameters $\alpha = 1.4$ and $\gamma = 0.45$, where this specific γ represents the waterfall region for such α. This comparison shows the convergence between the L_{abc} and the true LLR, and clearly show the improvement in term of LLR shapes compared to the demapper L_{ab}.

Moreover, Fig. 3 shows that the majority of the samples will fall within this part as can be seen from the pdf of the

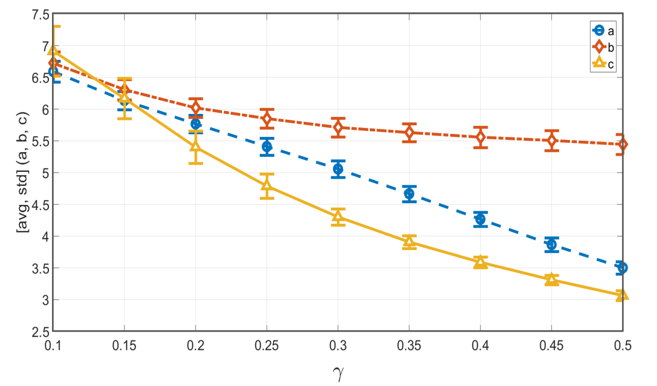

Fig. 6: Comparison of the mean and standard deviation evolution for parameter a, b and c as a function of the dispersion γ of a SαS noise with $\alpha = 1.4$ for the demapper L$_{abc}$.

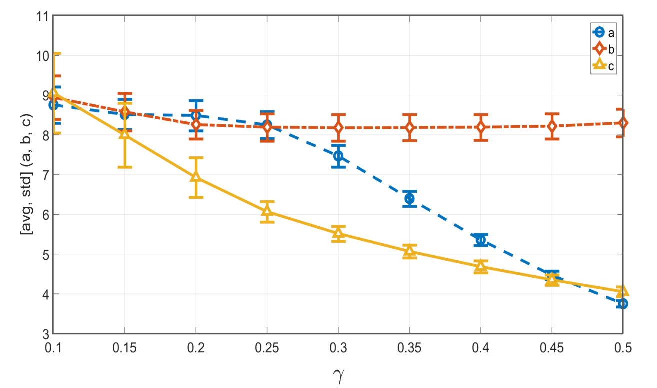

Fig. 7: Comparison of the mean and standard deviation evolution for parameter a, b and c as a function of the dispersion γ of a SαS noise with $\alpha = 1.8$ for the demapper L$_{abc}$.

received samples, for instance, for $\alpha = 1.4$ and $\gamma = 0.45$ the percentage error is (44$\pm$4)%. This percentage error is able to increase for different γ. In Fig. 5, we study the probability of receiving a sample within each region S_a (linear part), S_b (asymptotic part), or S_c (saturated part). For our simulation we use highly impulsive SαS noise where $\alpha = 1.8$. Fig. 5 shows that receiving a sample that falls within the transition phase depicted by the c parameter is highly probable. Furthermore, those samples highly influence the selection of the optimized parameters as well as the decoder performance in terms of BER and FER.

Fig. 6 , repsectively Fig. 7, compares the evolution of the mean and variance of the optimized parameters $\theta = (a, b, c)$ as a function of the dispersion γ, for less or more impulsive SαS noise, respectively. For each channel state, we ran 10000 experiments. The error bars indicate the small impact of different realizations in which we can infer the robustness of the parameter estimation method. Fig. 8 compares the shape of the true LLR, and the one obtained with the demapper L_{ab} and L_{abc}. Note that by adding a third parameter c, the obtained demapper L_{abc} allows a better fit to the true LLR than only with the two parameters describing the linear and asymptotic regions. Moreover, note that even in the linear and asymptotic parts, the fitting to the true LLR is better when

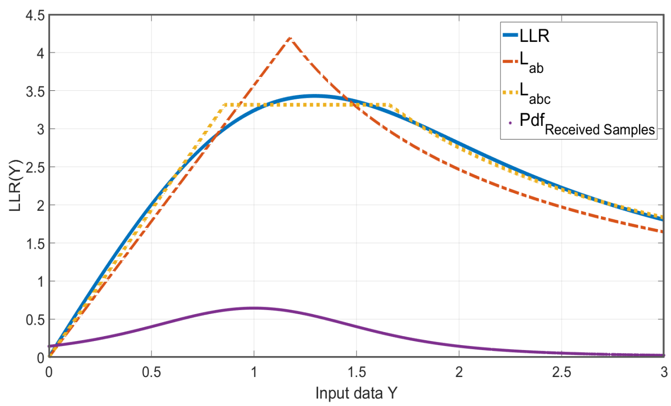

Fig. 8: Comparison of the LLR shapes obtained by numerical integration (true LLR) or with the approximation L_{ab} and L_{abc} under an $S\alpha S$ noise with $\gamma = 0.4$, $\alpha = 1.4$.

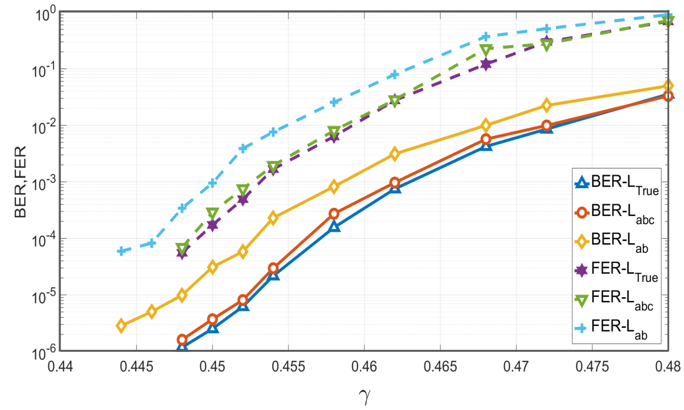

Fig. 10: Comparison of the BER and FER as a function of the dispersion γ of a SαS noise in more impulsive environment with $\alpha = 1.4$, between L_{ab}, L_{abc} and the LLR obtained by numerical integration.

three optimization parameters are used instead of only two. This can be justified by the fact that θ^* is subject of change depending on the pdf of the channel output Y. For instance, the samples that fall within the linear region will be represented by the optimal a. By using the L_{ab} demapper, we show that roughly half of the received samples will be treated as samples related to linear or asymptotic parts. Consequently, this will lead to a bad estimation compared to the L_{abc} demapping, where the c part releases the mismatch region sample selection. Eventually, each region will be depicted by a better estimated parameter that reflects the samples that fall in.

IV. Simulation Results

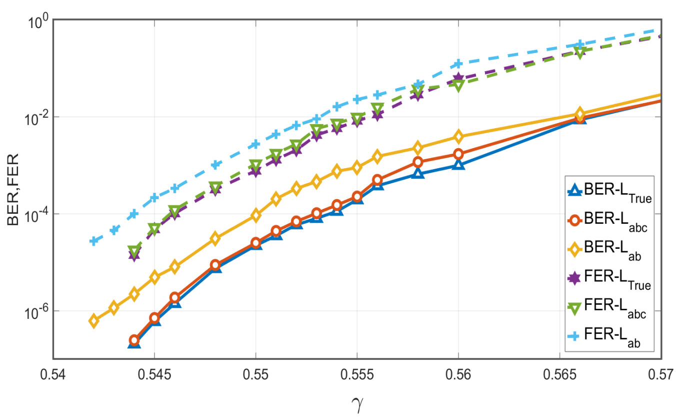

Fig. 9: Comparison of the BER and FER as a function of the dispersion γ of a SαS noise in less impulsive environment with $\alpha = 1.8$, between L_{ab}, L_{abc} and the LLR obtained by numerical integration.

To investigate the performance of different receivers, simulation results of Mackay's (3, 6) LDPC code with block size nb = 20000 bits are presented. The demapping function will be adjusted by the optimal parameters θ. In order to ensure a good estimation, we use a very long learning sequence of 20000 samples. The conventional noise power measurement is meaningless, since stable distributions do not have finite second-order moments when $\alpha \leq 2$ [11, Theorem 3]. Therefore, the following simulations will be presented as a function of the dispersion parameter γ, which is used as a measurement of the strength of the α-stable noise.

In Fig. 9 and Fig. 10 we present the BER and FER performance of different receivers for $\alpha = 1.8$ and $\alpha = 1.4$ which represent less and more impulsive channels, respectively. For each γ, the demapping functions L_{ab} and L_{abc} are compared with the optimal receiver. Our proposed solution improves the performance over the L_{ab} demapping function and matches the performance of the optimum receiver to the extent in both less and more impulsive channels, as it was expected, since the shape of the demapping function L_{abc} is quite the same as the shape of the true LLR as shown in Fig. 3.

V. Conclusion

In this paper, we proposed a new LLR demapper based on a parameterized approximation function with three optimization parameters. Having such an approximated LLR function is of crucial matter when the noise exhibits an impulsive nature and when the decoder relies on LLRs, such as LDPC, convolutional codes, turbo codes... Numerical simulations shown that the performance achieved with our proposed demapper match to the extent the one obtained with the true LLR and outperform the one obtained with previous proposed solutions. Moreover, our demapper feartures an easy implementation, whereas the true demapper is computationally burdened.

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