Optimal Power Allocation in a Relay-aided Cognitive Network

Anne Savard
IMT Lille Douai, Univ. Lille, CNRS, UMR 8520 - IEMN
F-59000 Lille, France
anne.savard@imt-lille-douai.fr

E. Veronica Belmega
ETIS UMR 8051, Université Paris Seine, Université de Cergy-Pontoise, ENSEA, CNRS
95000 Cergy, France
belmega@ensea.fr

ABSTRACT

In this paper, we address a power allocation problem in a relay-aided cognitive network. The network is composed by a primary and secondary user/destination pair and a relay, which helps the communication between the secondary user and its destination. The transmission of the secondary user and the helping relay is allowed provided that a minimum quality of service (QoS) constraint is met at the primary user. First, we derive the achievable rate regions under Decode-and-Forward (DF) and Compress-and-Forward (CF) relaying schemes. Then, we provide analytic expressions of the optimal power allocation policies at the secondary user and the relay. Remarkably, if the secondary direct link is negligible - the communication takes place only via the relay - DF is proven to always outperform CF, irrespective from the system parameters. If the secondary direct link is not negligible, our numerical results illustrate that DF outperforms CF only when the relay is close to the secondary user.

CCS CONCEPTS

• **Mathematics of computing** → **Coding theory**; **Convex optimization**; **Nonconvex optimization**;

KEYWORDS

Relay-aided cognitive network, Decode-and-Forward, Compress-and-Forward, Power allocation

ACM Reference Format:
Anne Savard and E. Veronica Belmega. 2019. Optimal Power Allocation in a Relay-aided Cognitive Network. In *12th EAI International Conference on Performance Evaluation Methodologies and Tools (VALUETOOLS 2019), March 12–15, 2019, Palma, Spain.* ACM, New York, NY, USA, 8 pages. https://doi.org/10.1145/3306309.3306314

1 INTRODUCTION

With the ever increasing number of communicating devices, one widely accepted solution for the next generation of wireless communication systems relies on cooperative communications [3]. Due to the nature of the wireless medium, all receivers within the range of a given transmitter can overhear its outgoing signals. Traditionally, the unintended signals were treated as additional noise, but in cooperative communications, one exploits these interfering signals to improve the network capacity [3].

The most basic model of such a cooperative communication is the relay channel, introduced by [23], where a relay helps the communication between a source and a destination. One key question is what operation the relay should perform in order to achieve the highest user's rate. Two main relaying schemes have been proposed in information theory: Decode-and-Forward (DF), where the relay decodes the message sent by the user and Compress-and-Forward (CF), where the relay quantizes the received signal [2]. Usually, none of these two relaying schemes perform best for all setups (in terms of the channel gains, noise variances, user's power, etc.). Nevertheless, these two relaying schemes have been shown to perform well over various extensions of the relay channel, such as: the *two-way relay channel* [12], where the relay helps a bi-directional communication between two users; the relay channel with *correlated noises* [16]; the *diamond relay channel* [15], where two relays help a bidirectional communication between two users; the *multi-way relay channel* [17], where a single relay helps multiple users grouped into clusters; and the *interference relay channel* [1, 14].

In this paper, we study a cognitive radio network, in which the secondary user is aided by a relay node operating in full-duplex mode. The objective is to find the optimal power control policies at both the secondary user and the relay node that maximize the opportunistic capacity while not disturbing *too much* the primary transmission. We assume that the primary user shares the spectrum with the secondary network and tolerates the resulting interference provided that it achieves a predefined target Shannon rate.

Resource allocation problems and in particular power allocation problems have been widely studied in cognitive radio networks [8–10, 19]. Game theory is employed in [19] to investigate the interaction between autonomous opportunistic users in a distributed multi-antenna (MIMO) cognitive radio (CR) network. In [8, 10] the centralized problem of joint scheduling and power allocation has been investigated in a multi-carrier CR network from a rate maximization and a power minimization perspective, respectively. In [9], an adaptive power allocation scheme is proposed in a dynamic MIMO CR network that can vary in an arbitrary and unpredictable manner. However, none of the aforementioned works investigate the impact of cooperative relaying on the performance of the secondary network.

Regarding relay-aided cognitive radio networks, there exists a vast literature on the topic: [5–7, 18, 21, 22, 24, 27, 28]. Several works investigate power allocation problems [5, 6] from an outage probability maximization perspective, whereas in this paper we are interested in the rate maximization problem. The authors of

VALUETOOLS 2019, March 12–15, 2019, Palma, Spain

ACM ISBN 978-1-4503-6596-3/19/03…$15.00
https://doi.org/10.1145/3306309.3306314

[7, 22, 24], investigate rate maximization problems in relay-aided cognitive radio networks and focus on Amplify-and-Forward relaying, whereas here we focus on two different relaying schemes, namely DF and CF. Another difference lies in the constraints imposed by the primary users; in [7, 22, 24], maximum peak interference constraints are considered, while we focus on a minimum Quality of Service (QoS) constraint.

The closest work to ours is [26], in which the authors investigate the energy-efficient maximization and rate maximization problems in a relay-aided cognitive radio network assuming DF relaying. The main differences between our work and [26] are three-fold. First, we study and compare two relaying schemes (DF and CF) instead of only DF. Second, our model is different in that we do not neglect the secondary direct link but we neglect the interfering link from the secondary user to the primary receiver, whereas in [26] the opposite case is investigated. Finally, we consider a QoS-based constraint imposed by the primary user as opposed to peak interference constraints.

The main contributions of this paper can be summarized as follows. First, we introduce a minimum Quality of Service (QoS) constraint to protect the primary user different than the more common maximum interference constraints [8], which allows the secondary user to transmit as long as the primary user achieves its desired target Shannon rate. Second, we derive the optimal power control policy at the secondary transmitter and the relay for two different relaying protocols: DF and CF. When the secondary direct link is negligible, DF provides always better results than CF, because of the lack of side information. If the secondary direct link is not negligible, DF outperforms CF only if the relay is close to the secondary user. At last, a suite of numerical simulations are provided to compare the performance results of the two protocols in various scenarios in function of the system parameters.

Although Amplify-and-Forward (AF) could be seen as a less complex alternative relaying scheme than DF and CF, we do not consider it in this work because of its poor performance in multi-user interference settings. Indeed, not only the useful signal and the noise are amplified but also all the interference terms. Adding to this the constraints imposed by the primary user, AF is unlikely to perform competitively with either DF or CF.

2 SYSTEM AND PROBLEM FORMULATION

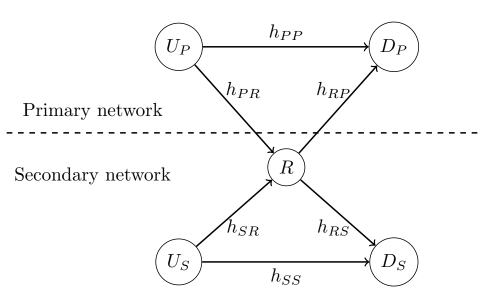

Figure 1: Cognitive relay-aided network

In this paper, we focus on a cognitive network depicted in Figure 1, where the primary network is composed by a primary user U_P and its associated destination D_P, whereas the secondary network is composed by a user U_S, its destination D_S and a relay R. The relay node operates in a full-duplex mode and can receive and transmit information at the same time and on the same frequency. We assume that perfect self-interference cancellation is performed at the relay. We also consider restricted encoders such that the nodes' inputs depend only of the current message and not on previously decoded symbols. For simplicity, the interfering links between the two user-destination pairs are assumed to be negligible. This situation can occur for instance when the user-destination pairs are located far away from one another.

Under the above assumptions, the received signals at the relay and at destination D_i, $i \in \{P, S\}$ are given as:

$$Y_R = h_{PR}X_P + h_{SR}X_S + Z_R, \tag{1}$$

$$Y_i = h_{Ri}X_R + h_{ii}X_i + Z_i, \tag{2}$$

where X_P, X_S and X_R are the transmitted signals of the primary user, secondary user and the relay, respectively; Z_R and Z_i, $i \in \{P, S\}$, model the effect of the Additive White Gaussian Noise (AWGN) at the relay and at destination D_i. Let N_R and N_i denote the variance of the received noises Z_R and Z_i. We also denote by P_P, P_S, and P_R the average powers of the input signals X_P, X_S, and X_R, respectively; and assume that the secondary user and the relay are power-constrained devices such that $P_S \leq \overline{P_S}$ and $P_R \leq \overline{P_R}$. Regarding the temporal sequence of message events, we consider a typical Block Markov coding such that, during each block k, the nodes receive and can process the messages sent during the previous block $k-1$.

The following notations will be used throughout the paper:

$$g_i = h_i^2, \quad i \in \{PP, PR, RP, SS, SR, RS\} \tag{3}$$

$$\mathcal{A} = \frac{g_{PP}P_P}{\left(1 + \frac{g_{PP}P_P}{N_P}\right)^{(1-\tau)} - 1}, \tag{4}$$

$$C(x) = \frac{1}{2}\log_2(1+x). \tag{5}$$

Let R_P denote the achievable rate of the primary user and $\overline{R_P}$ its single user achievable rate (in the absence of the secondary network), which can be easily computed as $\overline{R_P} = C\left(\frac{g_{PP}P_P}{N_P}\right)$. Let R_S denote the achievable rate of the secondary user.

The goal in this paper is to maximize the achievable secondary rate R_S under the following QoS constraint that protects the primary transmission:

$$R_P \geq (1-\tau)\overline{R_P}. \tag{6}$$

Intuitively, if the presence of the secondary user does not degrade the rate of the primary user more than a fraction $\tau \in [0, 1]$ of its (single user) initial rate, then the secondary user and the relay are allowed to transmit. If $\tau = 0$, no degradation is permitted by the primary user and the secondary relay is forced to stay silent. At the opposite, if $\tau = 1$, then the secondary network is always allowed to transmit irrespective from the harmful interference to the primary user.

To sum up, the optimization problem under study writes as

$$\begin{aligned} \underset{P_R, P_S}{\text{maximize}} \quad & R_S \\ \text{subject to} \quad & R_P \geq (1-\tau)\overline{R_P}, \\ & 0 \leq P_S \leq \overline{P_S}, \\ & 0 \leq P_R \leq \overline{P_R}. \end{aligned} \tag{7}$$

The remaining of the paper is organized as follows. In Section 3 and Section 4, we will first characterize the achievable rate region for two standard relaying schemes in Information Theory, Decode-and-Forward (DF) and Compress-and-Forward (CF). Based on these achievable rates, we will then rewrite our optimization problem and derive the optimal power control policies. In Section 5, we will compare the optimal performance obtained by these two relaying schemes. When the secondary direct link becomes negligible, we will analytically prove the superiority of the DF scheme over the CF scheme. In general this does not hold and our numerical results will illustrate different settings in which either DF or CF performs best. We will also question the utility of the relay. Finally, Section 6 concludes the paper and Section 7 contains our theoretical proofs.

3 DECODE-AND-FORWARD RELAYING

We start with the well known Decode-and-Forward relaying scheme. Since the relay is part of the secondary network, it considers the signal from the primary user as additional noise when decoding the secondary user's message. Similarly, since the primary destination is not interested in decoding the message from the secondary user, the signal received from the relay will be treated as additional noise. The following achievable rate region is obtained.

Proposition 3.1. *The following rate region is achievable over the cognitive relay-aided network, where the secondary relay employs a DF scheme and considers as noise the primary user's message:*

$$\bigcup_{0 \leq \alpha \leq 1} (R_P, R_S): \quad \begin{aligned} & R_P \leq C\left(\frac{g_{PP}P_P}{g_{RP}P_R + N_P}\right) \\ & R_S \leq C\left(\min\{f_{DF,1}(\alpha, P_S, P_R); f_{DF,2}(\alpha, P_S, P_R)\}\right), \end{aligned}$$

where $f_{DF,1}(\alpha, P_S, P_R)$ and $f_{DF,2}(\alpha, P_S, P_R)$ are given as

$$f_{DF,1}(\alpha, P_S, P_R) = \frac{g_{SR}(1-\alpha)P_S}{g_{PR}P_P + N_R} \text{ and}$$
$$f_{DF,2}(\alpha, P_S, P_R) = \frac{g_{SS}P_S + g_{RS}P_R + 2\sqrt{g_{RS}g_{SS}\alpha P_S P_R}}{N_S}.$$

The proof follows similarly to [4]. At the secondary user, superposition coding is used, where α allows to tradeoff the power between the repetition of the message sent in the previous block and sending a new message. The constraint on the primary rate follows by considering the perfect recovering of X_P at the primary destination by treating the message from the relay as additional noise. The two constraints on the secondary rate are obtained by performing a perfect recovering of X_S at the relay (by treating the message from the primary user as additional noise) and at the secondary destination.

From Proposition 3.1, if the relay performs DF, our optimization problem in (7) becomes

$$\begin{aligned} \underset{P_R, P_S, \alpha}{\text{maximize}} \quad & \min\{f_{DF,1}(\alpha, P_S, P_R);\ f_{DF,2}(\alpha, P_S, P_R)\} \\ \text{subject to} \quad & g_{RP}P_R \leq \mathcal{A} - N_P, \\ & 0 \leq P_S \leq \overline{P_S}, \\ & 0 \leq P_R \leq \overline{P_R}, \\ & 0 \leq \alpha \leq 1 \end{aligned} \tag{8}$$

The constraints are affine and define a convex feasible set. However, it can be easily checked that the objective function is not jointly concave w.r.t. (P_R, P_S, α) and, hence, the optimization problem above is not convex. Remarkably, we can still solve it analytically and in closed-form by exploiting the monotonicity properties of the functions $f_{DF,1}(\alpha, P_S, P_R)$ and $f_{DF,2}(\alpha, P_S, P_R)$.

Theorem 3.2. *If $N_P \geq \mathcal{A}$, no interference coming from the relay is tolerated at the primary destination and the solution of* (8) *is trivial: $P_R^* = 0, \alpha^* = 0$, and $P_S^* = \overline{P_S}$. The optimal achievable rate is $R_S^* = C\left(\min\left\{\frac{g_{SR}\overline{P_S}}{g_{PR}P_P + N_R}, \frac{g_{SS}\overline{P_S}}{N_S}\right\}\right)$.*

If $N_P < \mathcal{A}$, then the relay is allowed to transmit and the optimal powers are given as $P_S^* = \overline{P_S}$ and $P_R^* = \min\left\{\overline{P_R}, \frac{\mathcal{A} - N_P}{g_{RP}}\right\}$. The optimal choice of α depends on the system parameters:

a) if $\frac{g_{SR}P_S^*}{g_{PR}P_P + N_R} \leq \frac{g_{SS}P_S^* + g_{RS}P_R^*}{N_S}$, then $\alpha^* = 0$ and the optimal achievable rate equals $R_S^* = C\left(\frac{g_{SR}\overline{P_S}}{g_{PR}P_P + N_R}\right)$;

b) otherwise, $\alpha^* = \hat{\alpha}$, where $\hat{\alpha} \in (0, 1]$ is the unique intersection point between $f_{DF,1}$ and $f_{DF,2}$ w.r.t. $\alpha \in [0, 1]$; the optimal achievable rate equals $R_S^* = C\left(f_{DF,1}(\hat{\alpha}, \overline{P_S}, P_R^*)\right) = C\left(f_{DF,2}(\hat{\alpha}, \overline{P_S}, P_R^*)\right)$.

The detailed proof is given in Subsection 7.1.

Intuitively, if the primary destination is in poor conditions and the received noise is too high, then no additional interference from the secondary relay is tolerated. Otherwise, the relay is allowed to transmit. Also, since the achievable rate is increasing with the available powers P_S and P_R, the optimal solution is to transmit at the maximum powers, provided that the primary user meets its QoS constraint. Regarding α, the optimal solution is to balance the two opposing terms $f_{DF,1}$ and $f_{DF,2}$.

4 COMPRESS-AND-FORWARD RELAYING

Here, we assume that the relay performs Compress-and-Forward. Similarly to DF, since the primary user is not interested in decoding the message from the secondary user, the signal received from the relay will be treated as additional noise at the primary destination.

Proposition 4.1. *The following rate region is achievable over the cognitive relay-aided network, where the secondary relay employs a CF scheme and considers as noise the primary user's message:*

$$\bigcup (R_P, R_S): R_P \leq C\left(\frac{g_{PP}P_P}{g_{RP}P_R + N_P}\right)$$
$$R_S \leq C\left(P_S\left(\frac{g_{SR}}{g_{PR}P_P + N_R + D} + \frac{g_{SS}}{N_S}\right)\right),$$

where $D = \frac{P_S(g_{PR}g_{SS}P_P + g_{SR}N_S + g_{SS}N_R) + N_S(g_{PR}P_P + N_R)}{g_{RS}P_R}$.

Concerning the proof, the constraint obtained for the primary user's rate is obtained by considering a perfect recovering of X_P at the primary destination by treating the message from the relay as additional noise. The relay performs compression and the secondary message is recovered at the secondary destination using Wyner-Ziv coding with side information, similarly to [20].

We can rewrite the secondary user's rate as follows $R_S \leq C(f_{CF}(P_S, P_R))$ with

$$f_{CF}(P_S, P_R) = \frac{g_{SR} g_{RS} P_R P_S}{(g_{PR} P_P + N_R)(g_{RS} P_R + N_S + g_{SS} P_S) + g_{SR} P_S N_S} + \frac{g_{SS} P_S}{N_S}.$$

Using this reformulation, when the relay performs CF, our optimization problem in (7) reduces to

$$\begin{aligned} \underset{P_R, P_S}{\text{maximize}} \quad & f_{CF}(P_S, P_R) \\ \text{subject to} \quad & g_{RP} P_R \leq \mathcal{A} - N_P, \\ & 0 \leq P_S \leq \overline{P_S}, \\ & 0 \leq P_R \leq \overline{P_R} \end{aligned} \tag{9}$$

This optimization problem is not convex because of its non-concave objective, but the optimal solution can be obtained by investigating the monotonicity properties of the objective function, as in Section 3.

Theorem 4.2. *If $N_P \geq \mathcal{A}$, no interference from the relay is tolerated at the primary destination and the optimal power allocation of (9) is $P_R^* = 0, P_S^* = \overline{P_S}$ and the achievable rate equals $R_S^* = C\left(\frac{g_{SS}\overline{P_S}}{N_S}\right)$.*

If $N_P < \mathcal{A}$, then the optimal power allocation is given as $P_S^* = \overline{P_S}$ and $P_R^* = \min\left\{\overline{P_R}, \frac{\mathcal{A}-N_P}{g_{RP}}\right\}$. In this case, the optimal achievable rate equals $R_S^* = C\left(f_{CF}(\overline{P_S}, P_R^*)\right)$.

The proof follows via similar steps as in Theorem 3.2 and the details are omitted here. Notice that $f_{CF}(P_S, P_R)$ is an increasing function of P_R (for any fixed P_S) and is also increasing with P_S (for any fixed P_R). This means that, when the relay is allowed to transmit, then the optimal strategy is to transmit at maximum power while ensuring the primary user's QoS constraint: $P_S^* = \overline{P_S}$ and $P_R^* = \min\left\{\overline{P_R}, \frac{\mathcal{A}-N_P}{g_{RP}}\right\}$.

5 RELAYING SCHEMES COMPARISON

In this Section, we investigate which of the two protocols performs best and under which conditions. To this aim, we start with a simple case in which the secondary direct link is negligible, $g_{SS} = 0$, before moving to the general case.

5.1 Negligible secondary direct link ($g_{SS} = 0$)

We will first focus on the case where the direct link in the secondary network is negligible for simplicity. This can occur if the secondary user and its destination are located far apart.

We start by providing the achievable rate regions under DF and CF in this special case. The expressions can be easily derived by replacing $g_{SS} = 0$ in Proposition 3.1 and Proposition 4.1. Nevertheless, the coding schemes to achieve the rate regions are quite different when there is no direct link in the secondary network and, hence, the proofs behind these results cannot be derived from the proofs of Proposition 3.1 and Proposition 4.1.

Proposition 5.1. *The following rate region is achievable over the cognitive relay-aided network when the secondary direct link is negligible, when the relay employs a DF scheme:*

$$\bigcup (R_P, R_S) : R_P \leq C\left(\frac{g_{PP} P_P}{g_{RP} P_R + N_P}\right)$$
$$R_S \leq \min\left\{C\left(\frac{g_{SR} P_S}{g_{PR} P_P + N_R}\right), C\left(\frac{g_{RS} P_R}{N_S}\right)\right\}.$$

In the absence of the secondary direct link, no superposition coding is possible and the rate region above follows simply from point-to-point considerations and the fact that both the relay and the secondary destination must correctly decode their incoming signals (sent by the secondary user and the relay, respectively).

Proposition 5.2. *The following rate region is achievable over the cognitive relay-aided network when the secondary direct link is negligible, when the relay employs a CF scheme:*

$$\bigcup (R_P, R_S) : R_P \leq C\left(\frac{g_{PP} P_P}{g_{RP} P_R + N_P}\right)$$
$$R_S \leq C\left(f_{CF}^0(P_S, P_R)\right)$$

with

$$f_{CF}^0(P_S, P_R) = \frac{g_{SR} g_{RS} P_R P_S}{g_{RS} P_R (g_{PR} P_P + N_R) + N_S (g_{SR} P_S + g_{PR} P_P + N_R)}$$

The proof uses the following ingredients. The relay quantizes its observation based on nested lattices. The secondary destination first recovers the quantization index, estimates the observation of the relay without any side information, and then decodes the transmitted signal. The details are given in Subsection 7.2.

Now, since the secondary users' rates can be obtained by replacing $g_{SS} = 0$ in the general formulations (8) and (9), the solutions P_S^* and P_R^* can be easily found from Theorems 3.2 and 4.2. Note that for DF there is no superposition coding in this case and, thus, $\alpha = 0$.

What is remarkable is that we can rigorously prove that DF always outperforms CF in this particular case.

Theorem 5.3. *If the secondary direct link is negligible, the DF relaying scheme always outperforms CF in terms of secondary achievable rate.*

The proof is detailed in Subsection 7.3.

This result is rather surprising in the field of cooperative communications. For the Gaussian relay channel for instance, it is well known that DF outperforms CF if the relay is close to the user and DF is outperformed by CF when the relay is close to the destination. In our case, the CF scheme suffers from the absence of the direct link in the secondary network. This leads to the lack of any side information in the decoding step when estimating the signal received at the relay.

5.2 Non-negligible direct link ($g_{SS} > 0$)

In the particular case in which the secondary direct link is negligible, we have proved rigorously that the best relaying strategy is always DF. If the direct link is not negligible $g_{SS} > 0$, this result no longer holds and the best relaying scheme between DF and CF will depend on the system parameters. Providing an analytic answer is highly nontrivial and, hence, we provide below a numerical analysis instead.

The following setup will be considered in the remaining of the paper unless otherwise specified.[1] We assume that the nodes of our network are placed in a square cell of size 1×1. The position of the primary user and destination and of the secondary user and destination are fixed. The relay's position ranges over the cell. All channel gains are giving assuming a common path-loss model as $1/d^{3/2}$, where d is the distance between the two considered entities.

For the first setup, we further assume that $N_S = N_R = 1$, $N_P = 10$, and that the primary average power P_P and the peak powers $\overline{P_R}$ and $\overline{P_S}$ are all equal to 10. The coordinates of U_i, D_i, $i \in \{P, S\}$ are given as $U_P(0.1; 0.8)$, $D_P(0.2; 0.9)$, $U_S(0.1; 0.1)$, $D_S(0.9; 0.1)$.

Figure 2 shows a comparison between the secondary rates achieved with DF and CF for $\tau = 10\%$ - the primary user tolerates a 10% rate degradation. The light grey area corresponds to the set of relay's position where DF performs strictly better than CF, whereas the dark grey region corresponds to the set of relay's position where CF performs strictly better than DF. Similarly to the Gaussian relay channel, we remark that DF outperforms CF when the relay is close to the secondary user. Otherwise, CF outperforms DF.

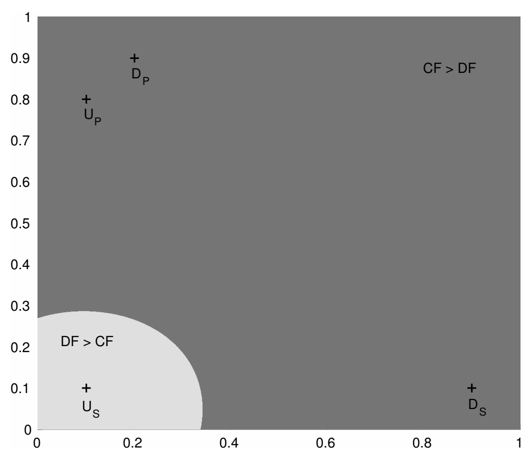

Figure 2: DF vs. CF comparison as a function of the relay position in the first setup. In the light gray area, DF outperforms CF, while in the dark gray area DF is outperformed by CF.

In the remaining of this Section, we investigate whether the relay is useful or harmful, when there is a non-negligible secondary direct link, or, otherwise stated, if the rate achieved with DF and CF can fall below the point-to-point rate (without the relay).

5.3 Is the relay always useful?

Without using the relay, the point-to-point achievable rate equals $C\left(\frac{g_{SS}\overline{P_S}}{N_S}\right)$. Here, the aim is to compare this rate with the ones obtained by using the relay, which are given in Proposition 3.1 and Proposition 4.1.

Proposition 5.4. *The achievable rate with the DF relaying scheme is always larger than the point-to-point one (over the direct link) if* $\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \geq \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S}$. *Otherwise, depending on the system parameters both cases arise:*
a) if $\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \leq \frac{g_{SS}P_S^*}{N_S} \leq \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S}$, *the relay is harmful;*
b) if $\frac{g_{SS}P_S^*}{N_S} \leq \frac{g_{SR}P_S^*}{g_{PR}P_P+N_R}$, *the relay is useful to the secondary user.*

The proof is given in Subsection 7.4.

The fact that the presence of the relay can harm the secondary user's rate comes from the DF scheme which imposes the relay to decode perfectly the transmitted signal. Hence, the quality of the link between U_S and R constrains the achievable rate. Otherwise stated, when the relay is in poor reception conditions, the secondary user has to transmit at a slow rate.

Figure 3 compares the secondary rate achieved by the DF scheme with the achieved rate without the relay. The following setup modifications are considered here: all noises are of unit variance, and $P_P = 10$, whereas $\overline{P_R} = \overline{P_S} = 1$. The coordinates of the users and destinations are given as: $U_P(0.1; 0.8)$, $D_P(0.2; 0.9)$, $U_S(0.6; 0.3)$, $D_S(0.9; 0.1)$. If the relay is close to the secondary user (the light gray area), then the DF scheme improves the achievable rate. In the dark grey areas, the presence of the relay harms the achievable rate compared to the point-to-point rate (without the relay). Note that the set of relay's positions such that DF outperforms CF is always included in the set of relay's positions such that DF improves the point-to-point rate.

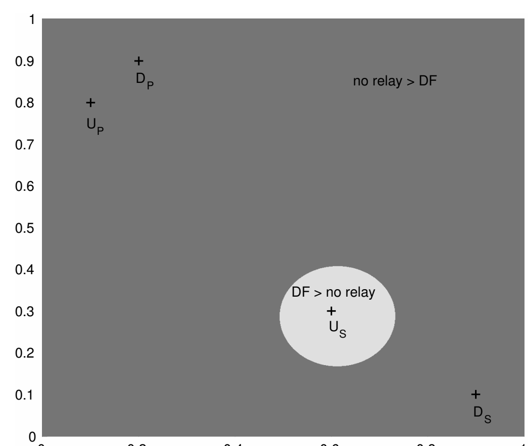

Figure 3: Df vs. no relay comparison as a function of the relay position. In the light gray area, the relay using DF is beneficial, while in the dark gray area the presence of the relay is harmful.

Proposition 5.5. *The achievable rate with the CF relaying scheme is always greater or equal to the point-to-point rate (without the relay).*

This result follows easily by inspecting the expression of f_{CF} in Section 4, in which the first additive term is missing without

[1] This numerical setup is chosen arbitrarily for illustrative purposes. We have conducted extensive simulations for different other settings and all our observations remain valid irrespective from the specific choice of the various parameters.

the relay. Hence, the presence of the relay always improves the secondary user's rate when CF scheme is employed.

Finally, Figure 4 compares the secondary rate achieved with CF and DF to the point-to-point rate (without the relay) as a function of $\tau \in [0,1]$ - the tolerance percentage of the primary user - for the setup: $N_R = N_S = N_P = 1$, $P_P = \overline{P_S} = \overline{P_R} = 10$, $U_P(0.1; 0.8)$, $D_P(0.2; 0.9)$, $U_S(0.2; 0.1)$, $D_S(0.5; 0.1)$, $R(0.25; 0.1)$. Note that the relay is placed close to the secondary user, thus, DF is expected to perform better than CF and/or than without the relay. At $\tau = 0$, the relay is not allowed to transmit and all rates are equal to the point-to-point case. Increasing τ means that the primary user tolerates higher rate degradation, which allows the relay to increase its transmit power resulting in higher secondary rates. The saturation regime is reached when the relay power equals the peak power constraint: $P_R^* = \overline{P_R}$. Also, the DF scheme performs better than CF, in this case.

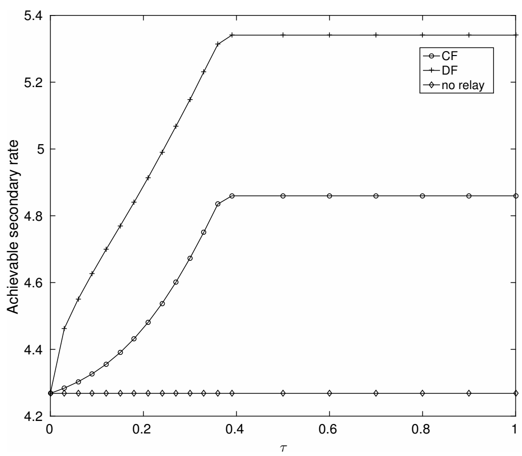

Figure 4: DF vs. CF vs. no relay comparison as a function of $\tau \in [0,1]$. When $\tau = 0$, the relay is not allowed to interfere at all with the primary destination and the achieved rates equal to the point-to-point rate. When τ increases, the tolerance of the primary user increases resulting in an increased transmit power at the relay. The saturation regime is reached when $P_R^* = \overline{P_R}$.

6 CONCLUSIONS AND PERSPECTIVES

A cognitive relay-aided network is investigated and the rate maximizing power allocation policies at the secondary user and its helping relay node are provided in this paper. We introduce a QoS-based constraint imposed by the primary user, which differs from the more common maximum interference constraints. Surprisingly, if the secondary user is far away from its destination, then DF relaying outperforms CF relaying, irrespective from the system parameters. Otherwise, DF only outperforms CF when the relay is close to the destination (similarly to the classic relay channel). Moreover, the relay is shown to be harmful when the DF scheme is employed and the relay is far away from the destination (in poor reception conditions), whereas for CF, the relay always improves the rate of the secondary user.

All our results carry over the more general setting in which the interfering link between the primary user and the secondary destination is not negligible, by simply adjusting the level of the received noise at the secondary destination. However, including the effect of the interfering link between the secondary user and the primary destination is non trivial and ongoing work. The main challenge is that the shape of the resulting QoS constraint becomes a non-convex function.

7 APPENDIX

7.1 Proof of Theorem 3.2

We distinguish two cases in function of the system parameters and the first constraint in (8) imposed by the primary user. First, if $N_P \geq \mathcal{A}$, $P_R^* = 0$ and the relay is not allowed to transmit. Now, by inspecting $f_{DF,1}$ and $f_{DF,2}$, it can easily be seen that $\alpha^* = 0$. Since for any fixed α and P_R both functions are increasing in P_S, the secondary transmitter should use its full power and $P_S^* = \overline{P_S}$.

Let us consider the non trivial case in which $N_P < \mathcal{A}$. Note that both functions $f_{DF,1}$ and $f_{DF,2}$ are increasing with P_R (for any α and P_S fixed) and with P_S (for any α and P_R fixed). Also, $f_{DF,1}$ is a decreasing function of α (for fixed P_R and P_S), whereas $f_{DF,2}$ is an increasing function of α (for any fixed P_R and P_S). Thus, in order to maximize both $f_{DF,1}$ and $f_{DF,2}$, the transmit powers P_S and P_R have to be set to their maximum allowed values. This leads to $P_S^* = \overline{P_S}$ and $P_R^* = \min\left\{\overline{P_R}, \frac{\mathcal{A}-N_P}{g_{RP}}\right\}$.

The remaining part of the proof is to find the optimal value of α. Since $f_{DF,1}$ is affine and decreasing in α and $f_{DF,2}$ is increasing in α, the optimal value of α depends of the relative position of both functions at $\alpha = 0$. Based on this, either $f_{DF,1}$ is always less than $f_{DF,2}$ and $\alpha^* = 0$ or there exists a unique intersection point lying in $[0,1]$ (because $f_{DF,1} = 0$ when $\alpha = 1$). To sum up, the optimal α^* is given as

$$\alpha^* = \begin{cases} 0, & \text{if } \frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \leq \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S} \\ \hat{\alpha}, & \text{otherwise} \end{cases} \tag{10}$$

where $\hat{\alpha} \in (0,1]$ is the unique intersection point of $f_{DF,1}$ and $f_{DF,2}$, or the solution to the equation $f_{DF,1}(\alpha, P_S^*, P_R^*) = f_{DF,2}(\alpha, P_S^*, P_R^*)$ w.r.t. $\alpha \in (0,1]$.

The analytic expression of $\widehat{\alpha}^2$ is given as

$$\widehat{\alpha}^2 = \frac{-K_1\sqrt{g_{SS}g_{RS}P_SP_R}}{g_{SR}N_SP_S} + \frac{\sqrt{K_2}}{g_{SR}N_SP_S}$$

with $K_1 = g_{PR}P_P + N_R$ and

$$K_2 = K_1^2 g_{SS}g_{RS}P_SP_R - g_{SR}N_SP_S\left((g_{SS}P_S + g_{RS}P_R)K_1 - g_{SR}N_SP_S\right).$$

7.2 Proof of Proposition 5.2

We start by briefly providing some necessary definitions and notions on lattice coding in order to derive the CF achievable rate region. For a full treatment on the topic, we refer the interested reader to [25].

A lattice Λ is a discrete additive subgroup of $\mathbb{R}^n$. Any point $x \in \mathbb{R}^n$ is nested to the nearest lattice point by the lattice quantizer Q_Λ as $Q_\Lambda(x) = \arg\min_{\lambda\in\Lambda} ||x - \lambda||$.

The *fundamental Voronoi region* $\mathcal{V}$ of the lattice Λ is the set of points that are closer to the origin than to any other lattice point:

$\mathcal{V} = \{x \in \mathbb{R}^n | Q_\Lambda(x) = 0\}$. The quantization error is obtained by the *modulo* Λ *operation*: $x \bmod \Lambda = x - Q_\Lambda(x)$. The *second moment per dimension* $\sigma^2(\Lambda)$ defines the average power of the lattice Λ: $\sigma^2(\Lambda) = \frac{1}{nV}\int_{\mathcal{V}} ||x||^2 dx$, where V is the volume of the fundamental Voronoi region of Λ.

Good lattice codebooks are build upon two nested lattices Λ and Λ_c, where $\Lambda \subseteq \Lambda_c$. These lattices are chosen such that Λ_c is Poltyrev [11]-good Λ is both Rogers [13]- and Poltyrev-good.

Encoding. Lattice-based codebooks are given as $C_i = \{\Lambda_{c_i} \cap \mathcal{V}_i\}$, where $\Lambda_i \subseteq \Lambda_{c_i}$ for $i \in \{S, Q, R\}$, where Q denotes the quantization. For $i \in \{S, R\}$, Λ_{ci} is Poltyrev-good and Λ_i is both Rogers- and Poltyrev-good whereas Λ_Q is Poltyrev-good and Λ_{cQ} is Rogers-good. In the remaining of proof, u_i, $i \in \{S, R, cQ\}$ is a dither uniformly distributed over $\mathcal{V}_i$.

The secondary user codebook is build upon nested lattices where we chose $\sigma^2(\Lambda_S) = P_S$ to ensure the secondary user average power constraint and Λ_{cS} such that $|C_S| = 2^{nR_S}$. During block b, the secondary user sends $c_S(b) \in C_S$ as

$$X_S(b) = [c_S(b) + u_S(b)] \bmod \Lambda_S.$$

The quantization codebook is build upon nested lattices where we chose $\sigma^2(\Lambda_{cQ}) = D$. The quantization rate is given as $R_q = \frac{1}{2}\log_2\left(\frac{\sigma^2(\Lambda_Q)}{D}\right)$, where $\sigma^2(\Lambda_Q)$ will be specified later on in the proof. The relay codebook is build upon lattices where we chose $\sigma^2(\Lambda_R) = P_R$ to ensure the secondary user average power constraint. Each compression index $i \in C_q$ is mapped to one codeword $c_R \in C_R$, that is Λ_R is chosen s.t. $|C_R| = 2^{nR_q}$.

During block b, the relay sends

$$X_R(b) = [c_R(I(b-1)) + u_R(b)] \bmod \Lambda_R.$$

Decoding. The quantization index during block b at the relay is computed as

$$I(b) = [Q_{cQ}(\beta Y_R(b) + u_{cQ}(b)] \bmod \Lambda_Q \tag{11}$$
$$= [\beta Y_R(b) + u_{cQ}(b) + E_q(b)] \bmod \Lambda_Q, \tag{12}$$

where E_q is the quantization error and β a scaling factor that will be specified later on in the proof.

During block b, the primary destination receives $Y_P(b) = h_{PP}X_P(b) + h_{RP}X_R(b) + Z_P(b)$ and recovers X_P by treating the message from the relay as additional noise as long as $R_P \leq C\left(\frac{g_{PP}P_P}{g_{RP}P_R+N_P}\right)$.

During block b, the secondary destination receives $Y_S(b) = h_{RS}X_R(b) + Z_S(b)$. It starts by recovering the quantization index, which is possible as long as $R_q \leq C\left(\frac{g_{RS}P_R}{N_S}\right)$.

It then estimates the received signal at the relay as

$$\widehat{Y}_R(b) = \beta[I(b) - u_{cQ}(b)] \bmod \Lambda_Q$$
$$\overset{(a)}{=} \beta^2 Y_R(b) + \beta E_q(b)$$

where (a) requires that $\sigma^2(\Lambda_Q) \geq \beta^2(g_{SR}P_S + g_{PR}P_P + N_R) + D$.

Finally, using $\widehat{Y}_R$, the secondary destination recovers X_S as long as $R_S \leq C\left(\frac{\beta^2 g_{SR}P_S}{\beta^2(g_{PR}P_P+N_R)+D}\right)$.

In order to satisfy a maximum distortion of D, β is chosen as $\beta^2 = 1 - \frac{D}{g_{SR}P_S+g_{PR}P_P+N_R}$. This choice of β^2, yields the following expression for $\sigma^2(\Lambda_2)$: $\sigma^2(\Lambda_2) = g_{SR}P_S + g_{PR}P_P + N_R$. Since $R_q = \frac{1}{2}\log_2\left(\frac{\sigma^2(\Lambda_2)}{D}\right) \leq C\left(\frac{g_{RS}P_R}{N_S}\right)$, D is chosen as $D = \frac{N_S(g_{SR}P_S+g_{PR}P_P+N_R)}{g_{RS}P_R+N_S}$ since the smallest distortion yields the largest achievable rate.

7.3 Proof of Theorem 5.3

Let us consider separately the two cases that can arise for DF. First, we assume that $\min\left\{\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R}; \frac{g_{RS}P_R^*}{N_S}\right\} = \frac{g_{SR}P_S^*}{g_{PR}P_P+N_R}$. One can show that $f_{CF}^0 - \frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \leq 0$. This implies that CF is outperformed by DF in terms of achievable rate in this case.

Second, if $\min\left\{\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R}; \frac{g_{RS}P_R^*}{N_S}\right\} = \frac{g_{RS}P_R^*}{N_S}$, we can show that $f_{CF}^0 - \frac{g_{RS}P_R^*}{N_S} \leq 0$, which implies here as well that CF is outperformed by DF.

7.4 Proof of Proposition 5.4

For DF, two cases can arise depending on the sign of the expression: $\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} - \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S}$.

If $\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \geq \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S}$, then the optimal α in Proposition 3.1 equals $\alpha^* = \hat{\alpha}$ and the achievable secondary rate is given as $R_S^* = C\left(\frac{g_{SS}P_S^*+g_{RS}P_R^*+2\sqrt{g_{RS}g_{SS}\hat{\alpha}P_S^*P_R^*}}{N_S}\right)$. Since the following inequalities hold

$$\frac{g_{SS}P_S^*}{N_S} \leq \frac{g_{SS}P_S^* + g_{RS}P_R^*}{N_S} < \frac{g_{SS}P_S^* + g_{RS}P_R^* + 2\sqrt{g_{RS}g_{SS}\alpha_1 P_S^* P_R^*}}{N_S},$$

the presence of the relay improves the achievable rate of the secondary user.

If $\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R} \leq \frac{g_{SS}P_S^*+g_{RS}P_R^*}{N_S}$, then the optimal α in Proposition 3.1 equals $\alpha^* = 0$ and the achievable secondary rate is given as $R_S^* = C\left(\frac{g_{SR}P_S^*}{g_{PR}P_P+N_R}\right)$. In this case, one can have either

$$\frac{g_{SS}P_S^*}{N_S} \leq \frac{g_{SR}P_S^*}{g_{PR}P_P + N_R} \leq \frac{g_{SS}P_S^* + g_{RS}P_R^*}{N_S},$$

leading to an increased achievable rate with help of the relay, or

$$\frac{g_{SR}P_S^*}{g_{PR}P_P + N_R} \leq \frac{g_{SS}P_S^*}{N_S} \leq \frac{g_{SS}P_S^* + g_{RS}P_R^*}{N_S},$$

leading to a worse achievable rate than the point-to-point one (without the relay).

ACKNOWLEDGEMENT

This work was partly supported by IRCICA, CNRS USR 3380, Lille, France.

REFERENCES

[1] E. V. Belmega, B. Djeumou, and S. Lasaulce. 2010. Power allocation games in interference relay channels: Existence analysis of nash equilibria. *EURASIP Journal on Wireless Communications and Networking* 2010 (2010), 114.
[2] T. M. Cover and A. E. El Gamal. Sept. 1979. Capacity theorems for the relay channel. *IEEE Transactions on Information Theory* 25, 5 (Sept. 1979), 572–584.
[3] Mingjun Dai, Peng Wang, Shengli Zhang, Bin Chen, Hui Wang, Xiaohui Lin, and Cong Sun. 2014. Survey on cooperative strategies for wireless relay channels. *Transactions on Emerging Telecommunications Technologies* 25, 9 (2014), 926–942.
[4] A. Khina, O. Ordentlich, U. Erez, Y. Kochman, and G. W. Wornell. 2012. Decode-and-forward for the Gaussian relay channel via standard AWGN coding and decoding. *IEEE Information Theory Workshop (ITW), Lausanne, Sweiss* (2012).

[5] H. Kim, S. Lim, H. Wang, and D. Hong. 2012. Optimal power allocation and outage analysis for cognitive full duplex relay systems. *IEEE Transactions on Wireless Communications* 11, 10 (2012), 3754–3765.
[6] J. Lee, H. Wang, J. G. Andrews, and D. Hong. 2011. Outage probability of cognitive relay networks with interference constraints. *IEEE Transactions on Wireless Communications* 10, 2 (2011), 390–395.
[7] L. Li, X. Zhou, H. Xu, G. Y. Li, D. Wang, and A. Soong. 2011. Simplified relay selection and power allocation in cooperative cognitive radio systems. *IEEE Transactions on Wireless Communications* 10, 1 (2011), 33–36.
[8] R. Masmoudi, E. V. Belmega, and I. Fijalkow. 2016. Efficient spectrum scheduling and power management for opportunistic users. *EURASIP Journal on Wireless Communications and Networking* 2016, 1 (2016), 97.
[9] P. Mertikopoulos and E. V. Belmega. 2014. Transmit without regrets: online optimization in MIMO-OFDM cognitive radio systems. *IEEE Journal on Selected Areas in Communications* 32, 11 (2014), 1987–1999.
[10] D. T. Ngo and T. Le-Ngoc. 2011. Distributed resource allocation for cognitive radio networks with spectrum-sharing constraints. *IEEE Transactions on Vehicular Technology* 60, 7 (2011), 3436–3449.
[11] G. Poltyrev. Mar. 1994. On coding without restrictions for the AWGN channel. *IEEE Trans. on Information Theory* 40, 2 (Mar. 1994), 409–417.
[12] B. Rankov and A. Wittneben. 2006. Achievable rate regions for the two-way relay channel. *IEEE International Symposium Information Theory (ISIT)* (2006).
[13] C. A. Rogers. 1959. Lattice coverings of space. *Mathematica* 6 (1959), 33–39.
[14] Onur Sahin and Elza Erkip. 2007. Achievable rates for the Gaussian interference relay channel. In *Global Telecommunications Conference, 2007. GLOBECOM'07. IEEE.* 1627–1631.
[15] A. Savard and L. Clavier. 2018. On the two-way diamond relay channel with lattice-based Compress-and-Forward. *IEEE Wireless Communications and Networking Conference (WCNC)* (2018).
[16] A. Savard and C. Weidmann. 2015. Lattice coding for the Gaussian one- and two-way relay channels with correlated noises. *IEEE International Symposium Information Theory (ISIT)* (2015).
[17] A. Savard and C. Weidmann. 2016. On the Gaussian multiway relay channel with intra-cluster links. *EURASIP Journal on Wireless Communications and Networking, SpringerOpen* 52 (2016), 1–17.
[18] A. Scaglione, D. L. Goeckel, and J. N. Laneman. 2006. Cooperative communications in mobile ad hoc networks. *IEEE Signal Processing Magazine* 23, 5 (2006), 18–29.
[19] G. Scutari and D. P. Palomar. 2010. MIMO cognitive radio: A game theoretical approach. *IEEE Transactions on Signal Processing* 58, 2 (2010), 761–780.
[20] Y. Song and N. Devroye. Aug. 2013. Lattice codes for the Gaussian relay channel: Decode-and-forward and Compress-and-Forward. *IEEE Transactions on Information Theory* 59, 8 (Aug. 2013), 4927 – 4948.
[21] S. Sridharan, S. Vishwanath, S. A. Jafar, and S. Shamai. 2008. On the capacity of cognitive relay assisted Gaussian interference channel. *IEEE International Symposium Information Theory (ISIT)* (2008).
[22] L. T. Tan, L. Ying, and D. W. Bliss. 2017. Power allocation for full-duplex relay selection in underlay cognitive radio networks: Coherent versus non-coherent scenarios. *arXiv preprint arXiv:1703.01527* (2017).
[23] E. C. van der Meulen. 1971. Three-terminal communication channels. *Adv. Appl. Prob.* 3 (1971), 120–154.
[24] M. Wu, J. Lou, D. Liu, T. Luo, and G. Yue. 2010. Power allocation for cognitive relay networks. In *Network Infrastructure and Digital Content, 2010 2nd IEEE International Conference on.* 41–45.
[25] R. Zamir. 2009. Lattices are everywhere. *Proc. Inf. Theory Appl. Wkshp., La Jolla, CA* (2009), 392–421.
[26] Z. Zhang, W. Qihui, and W. Jinlong. 2012. Energy-Efficient Power Allocation Strategy in Cognitive Relay Networks. *Radioengineering* 21, 3 (2012).
[27] G. Zhao, C. Yang, G. Y. Li, D. Li, and A. C. Soong. 2011. Power and channel allocation for cooperative relay in cognitive radio networks. *IEEE Journal of Selected Topics in Signal Processing* 5, 1 (2011), 151–159.
[28] Y. Zou, Y. D. Yao, and B. Zheng. 2012. Cooperative relay techniques for cognitive radio systems: Spectrum sensing and secondary user transmissions. *IEEE Communications Magazine* 50, 4 (2012).