Blind Estimation of an Approximated Likelihood Ratio in Impulsive Environment

Yasser Mestrah*†, Anne Savard*, Alban Goupil†, Laurent Clavier* and Guillaume Gellé†
*IMT Lille Douai, Univ. Lille, CNRS, UMR 8520 - IEMN, F-59000 Lille, France
†CReSTIC, University of Reims Champagne-Ardenne, France

***Abstract*—Robust communication is necessary for many wireless applications. Making a decision at the receiver requires an evaluation of the likelihood. However, in impulsive noise, the traditional Gaussian-based receiver exhibits a very significant performance loss. This paper proposes to approximate the likelihood ratio in a binary transmission with a function adapted to impulsive noise conditions but also efficient when noise is purely Gaussian. We introduce a blind estimation of the two parameters defining the approximation and evaluate its performance when used as the inputs of the belief propagation decoder. Our proposal allows us not only to achieve performance close to the optimal decoding but also to have a simple implementation and to adapt to different environment, impulsive or not, independently of the underlying statistical noise model, without the need of a training sequence.**

***Index Terms*—Soft iterative decoding, impulsive interference, impulsive noise, alpha-stable distribution, supervised learning, unsupervised learning.**

I. Introduction

Low-Density Parity Check (LDPC) codes, introduced by Gallager [1] and rediscovered by MacKay are a powerful linear block coding tool, since the sparseness of their parity check matrices guaranties a decoding complexity that increases quasi-linearly with the code length. They are widely used in various standards, as for example in wireless local area networks (WLAN IEEE 802.11n), Digital video broadcasting and Digital audio broadcasting. Usually, LDPC codes are decoded with the Belief Propagation (BP) algorithm, relying on Log-Likelihood Ratio (LLR) and perform near the capacity under Gaussian noise.

However, in many communication settings, noise cannot be modeled by a Gaussian distribution since it exhibits an impulsive behavior [2] [3]. Many works have tackled the modelization question and numerous solutions have been proposed since the first works by Middleton [4]. Recently, stochastic geometry has been used to address the problem in different network scenarios, but the analytical expression of the noise density is often complex (infinite series) or non tractable (α-stable). Consequently, designing the optimal receiver is complex, even assuming we can have a perfect knowledge of the interference statistics.

If we use a traditional receiver, relying on the Gaussian noise assumption, the decoding will suffer from severe performance degradation, due to the noise model mismatch, not being able to handle the strong impact of impulsive samples. Several approaches have been proposed in the literature to improve the robustness of a receiver against impulsive noise. For instance an approximation of the interference distribution can be proposed which leads to an analytical receiver design (for instance a generalized Gaussian distribution approximation in [6], a mixture of Laplacian and Gaussian in [5], a Cauchy distribution in [3]), a Normal Inverse Gaussian in [7]). To be less specific on a noise model, some robust metrics to estimate distance have also been used, for instance in [9] a saturation of the metric obtained in the Gaussian case (not of the received sample, unlike the soft-limiter), the p-norm in [8] or the Hubber metric in [10]. We proposed in [11] to directly approximate the LLR that will serve as input of the BP algorithm for LDPC codes, moreover, our solution is not limited to these codes. Choosing a family of function for this approximation in a space defined by a small set of parameters allows both to estimate those parameters with a limited complexity and to adapt to different noise statistical properties, impulsive or not.

We propose to approximate the LLR with parametric function chosen in the set $f(x) = \text{sign}(x)\min(a|x|, b/|x|)$. The main contribution of our paper is to propose a simple, fast and easy way to implement a blind estimation of the parameters a and b. It brings two main advantages over the previously proposed approaches:

1) It avoids the use of a training sequence which reduces the useful rate of the transmission.
2) It allows to use the full packet for the estimation which can be a great advantage especially when large samples are not so frequent and could be missing from the training sequence, misleading the estimation process.

The rest of this paper is organized as follows: The system model is presented in Section II. Section III starts, for the sake of completeness, by presenting the supervised LLR approximation with mutual information criterion. We then propose our unsupervised LLR approximation and provide a performance comparison with the supervised approximation. Section IV gives some simulation results using a regular LDPC code under

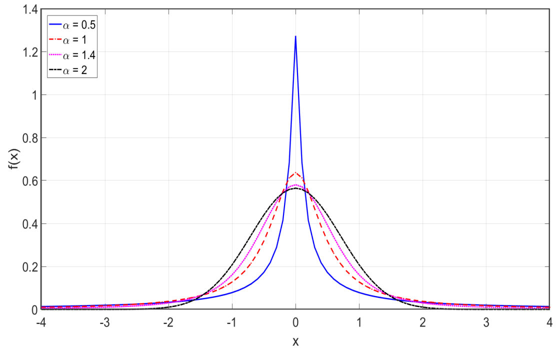

Fig. 1. Pdfs of SαS distributions ($\gamma = 0.5$).

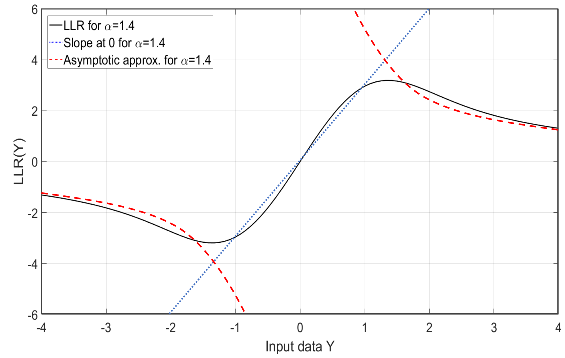

Fig. 2. LLR demapper for α=1.4, $\gamma = 0.5$, and its approximation.

impulsive noise and finally, Section V concludes the paper.

II. System Model and Background

In this paper, we consider the transmission of a binary message X in presence of interference. Let Y denotes the received message modeled by $Y = X + N$, where N denotes the interference, that is assumed to be independent of X. Throughout the paper, we assume that the information source X belongs to a simple binary phase-shift keying (BPSK) constellation where $X = \pm 1$ with equal probability and N follows a symmetric alpha stable ($S\alpha S$) distribution.

The characteristic function of a $S\alpha S$ random variable is given as $\phi_{S_\alpha}(t) = \exp\left(-|\gamma t|^\alpha\right)$, where $(0 < \alpha \leq 2)$ is the characteristic exponent and γ the dispersion. In wireless context, α is directly associated with the path loss exponent of the radio channel [12].

Fig. 1 shows the pdf of various α-stable distributions associated with different values of α. Remark that the smaller the value of α, the heavier the tail of the pdf, which increases the likelihood of having impulses with large amplitudes and far from the center location. The dispersion measures the spread of the noise and is considered as a scale parameter, similarly to the variance for Gaussian distribution, which is a special case of SαS distribution with $\alpha = 2$ and $\gamma = \sigma/\sqrt{2}$.

In this paper, we assume that X is encoded using LDPC codes, whose parity check matrix H, of dimension $m \times n$, is sparse, i.e. has a low density of ones. Their usually associated decoder, the Belief Propagation (BP) algorithm, expects the input to be given as LLRs. Thus, in order to use the BP decoder, one has to transform the channel output Y into LLR, which is thereafter called demapping. Consequently, working on this demapping allows our solution to be compatible with all types of decoders. The decoder inputs are given as:

$$LLR(y) = \log\left(\frac{\Pr[Y = y|X = +1]}{\Pr[Y = y|X = -1]}\right) = \log\left(\frac{f_N[y-1]}{f_N[y+1]}\right),$$

where $f_N(\cdot)$ is the pdf of the noise N.

If $\alpha = 2$, the studied channel reduces to the AWGN channel. In this case, one has access to a closed-form expression of the noise pdf. The decoder inputs are given as $LLR_{\text{Gauss}}(y) = \frac{2y}{\sigma^2} = \frac{y}{\gamma^2}$. This is the widely used LLR demapper, which is a linear function of y, which slope depends only on the channel conditions.

However, in the general case, $S\alpha S$ distributions does not have a closed-form expression for the pdf. We can nevertheless overcome this impediment by numerical calculation of the LLR e.g. by numerical integration of the inverse Fourier transform of the characteristic function, as $f_{\alpha,\gamma}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-|\gamma t|^\alpha) e^{-itx}\, dt$. However, this induces a generally prohibitive computational burden as well as requiring the knowledge of the noise parameters. Fig. 2 illustrates the non-linearity of the LLR function for the channel output y when the noise is α-stable with $\alpha = 1.4$ and $\gamma = 0.5$. Except for $\alpha = 2$, this shape is similar for every value of α. Fig. 2 clarifies that, as the output channel increases, the LLR decreases meaning that the received sample becomes less reliable. Moreover, two specific parts can be observed: when the output is close to zero, the LLR is almost a linear function of y, whereas when y is larger the LLR presents a power-law decrease. This observation is the starting point of our supervised approximated demapper used for LDPC codes under additive impulsive noise [11]. Even if Fig. 2 delineates a special impulsive noise model, the overall appearance of the LLR stays the same for the other noise models.

In the next section, we start by presenting our supervised LLR approximation as proposed and we then extend it yielding a novel unsupervised approximated LLR demapper.

III. Proposed demapper based on approximated LLR

To avoid the complexity induced by the α-stable assumption about the noise and to have a solution robust to a model mismatch, we propose to implement a simple approximated demapper.

Based on the presence of the two aforementioned parts in the LLR, we proposed in [11] the following LLR approximation:

$$L_\theta(y) = \begin{cases} ay & \text{if } |y| < \sqrt{b/a}\ , \\ b/y & \text{otherwise.} \end{cases} \tag{1}$$

which requires the knowledge of the parameter $\theta = (a, b)$, a>0 and b>0, to tune the receiver, and thus to match to the channel situation. In [13] we proposed to estimate θ in a supervised manner, which can drastically decrease the throughput, because of the needed learning sequence. In this paper, we propose to optimize θ in an unsupervised manner.

In [13], we proposed three methods solving the supervised optimization : two of them were dependent on the noise model, whereas the last one was based on a maximization of the mutual information (MI) between a learning sequence and the approximated demapper. We showed that the later yields the best performance in terms of bit error rate (BER) in various impulsive noise types.

In this paper, we propose to perform the blind optimization of the demapper using the MI based method.

The channel capacity has been well studied in the literature [14]. For memoryless binary input symmetric-output channel (MBISO) that corresponds to our channel model, the capacity is given by the MI between the input X and the channel output Y as $C = I(X, Y)$, where the binary input is uniformly distributed. As a type of MBISO, the mutual information over the additive SαS noise channel can be expressed as:

$$I_L(X, Y) = 1 - \mathbb{E}\left[\log_2(1 + e^{-XL(Y)})\right], \tag{2}$$

where L denotes the LLR. The MI with approximated LLR L_θ is thus given as:

$$I_{L_\theta}(X, Y) = 1 - \mathbb{E}\left[\log_2(1 + e^{-XL_\theta(Y)})\right]. \tag{3}$$

Authors in [15] proved that (3) reaches its maximum when the pdf of L_θ is equal to the pdf of true L.

Theoretically, to find back the optimum LLR from the MI we must maximize $I_{L_\theta}(X; Y)$. In order to narrow the search space of the best function that fits to the optimum L, we look for the parametrized function L_θ as we just proposed. Thus, to fit the optimal L our goal is to find θ that maximizes the mutual information as:

$$\theta^* = \arg\max_\theta I_{L_\theta}(X; Y) \tag{4}$$

The expectation operator in (3) relies on the noise distribution knowledge. Since we don't make any assumption on the noise model, the knowledge of the noise distribution is missed, but a good estimation is authorized by replacing the expectation operator by an empirical average with large values of N. Our optimization problem can thus be rewritten as:

$$\begin{aligned} \theta^* &\approx \arg\max_\theta 1 - \frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + e^{-x_n L_\theta(y_n)}\right) \\ &\approx \arg\min_\theta \underbrace{\frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + e^{-x_n L_\theta(y_n)}\right)}_{f_{\text{opt}}(x_n;\, y_n)}, \end{aligned} \tag{5}$$

where x_n and y_n are samples that represent the input and output of the channel respectively.

The minimization of $f_{\text{opt}(\cdot)}$ will be tackled in our implementation via simplex method based algorithm [16]. Eventually, the goal is to optimize the parameter θ, which allows us to find back the best approximated LLR that fits the optimum L.

To get the samples x_n and y_n two ways are considered: the supervised way, where x_n is a learning sequence and y_n the output of the channel given the input x_n and the unsupervised way, where only y_n is known to the receiver which needs thus to rebuilt from y_n a corresponding input sequence x_n.

In the following, we will start by briefly presenting the supervised optimization as performed in [11], and then, we will present our proposed unsupervised optimized demapper.

A. Supervised

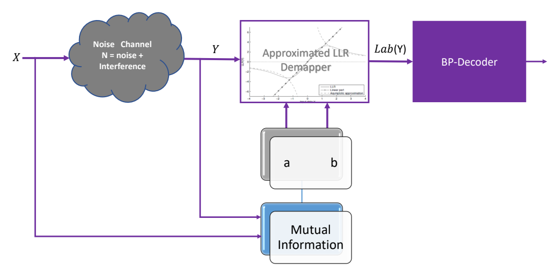

Fig. 3. Supervised LLR demapper.

If the optimization is performed in a supervised manner, the value of the parameter $\theta = (a, b)$ is triggered after receiving the channel output Y by maximizing the MI between the learning sequence X and channel output Y as shown in Fig. 3.

The major drawback of this method is the large needed length of the learning sequence, inducing a tedious overload for each packet transmission. Because of the impulsive nature of the noise, the estimation is more complex than in the Gaussian case. Indeed, the learning sequence has to be long enough to guaranty the presence of large impulse amplitude, else the estimation of θ won't lead to a good LLR estimation.

In the next subsection, we propose to overcome these issues by providing a blind estimation of a and b, which focuses directly on the output of the channel, without any prior on the input.

B. Unsupervised

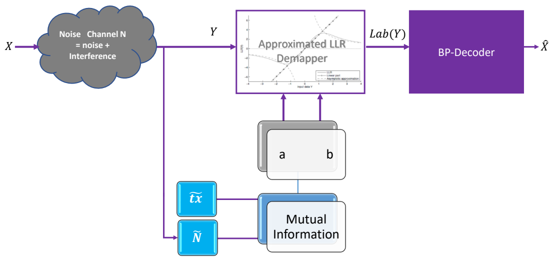

Fig. 4. Unsupervised LLR demapper.

Unsupervised optimization is notably attractive since it does not rely on any overload.

Since the MI based method requires the noise knowledge in order to optimize θ, we propose to estimate noise samples directly from the received samples Y using a sign detector as $\widetilde{N} = Y - \text{sign}(Y)$. Once the noise samples are extracted, one can simulate a new channel for which the input sequence $\widetilde{X}$ is known and follows an i.i.d. BPSK independent of Y and $\widetilde{N}$. The output of this new channel will be given as $\widetilde{Y} = \widetilde{X} + \widetilde{N}$. Since this known sequence is designed at the decoder, the method will not suffer from a throughput loss: even if $\widetilde{X}$ can be seen as a learning sequence used to mimic the supervised optimization, it is not sent over the channel. The optimal value of the parameter θ is obtained by maximizing the MI between the known input $\widetilde{X}$ and the output of the designed channel $\widetilde{Y}$ using approximated LLRs. Once the optimal parameter θ^* is obtained over the designed channel, it is used to build the approximated demapper over the real channel as $L_{\theta^*}(Y)$.

C. Comparison between the supervised and unsupervised optimization

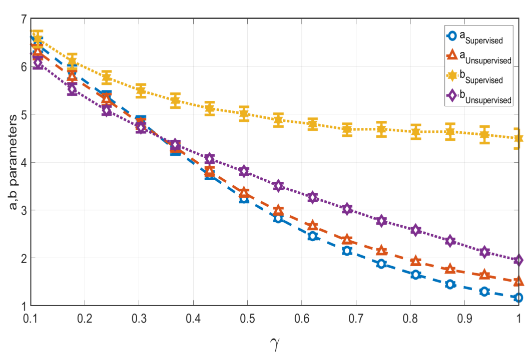

Fig. 5. Comparison of the mean and standard deviation evolution for parameters (a, b) as a function of the dispersion γ of a SαS noise with $\alpha = 1.4$ for the supervised and unsupervised optimization.

In order to be efficient, we can expect that the approximated demapper under blind optimization performs close to the one using a learning sequence. Thus, the optimal values for θ obtained under the supervised and the unsupervised optimization must be close to each other.

Under both supervised and unsupervised optimization, the evolution of the mean and variance of the parameter $\theta = (a, b)$ are compared as a function of the dispersion γ of a $S\alpha S$ noise with $\alpha = 1.4$ as shown in Fig. 5. For each channel state, a and b are resultant of 1000 experiments, furthermore, a learning sequence of 20000 samples is used to optimize a and b under the supervised case. Such a long sequence allows to limit estimation errors so that a and b will be obtained with a high accuracy in the supervised approach. Whereas the gap between the obtained values for a, under supervised and unsupervised optimization, is rather small, the one obtained for b is significantly larger. Nevertheless, this will not have major consequences in terms of BER performance as we will see in the next section. Fig. 6

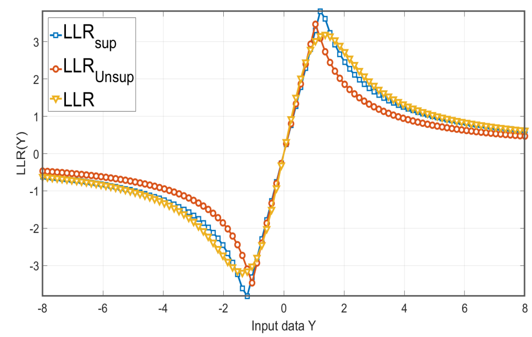

Fig. 6. Comparison of the LLR shapes under the effect of the estimated a and b parameters with $\gamma = 0.5$ and $\alpha = 1.4$, in the supervised LLR approximation, the unsupervised LLR approximation and with LLR obtained by numerical integration.

compares the LLR shapes obtained under supervised optimization ($a = 3.15, b = 4.96$) and unsupervised optimization ($a = 3.3, b = 3.75$) to the true LLR obtained via numerical integration for a $S\alpha S$ noise of parameters $\alpha = 1.4$ and $\gamma = 0.5$. This comparison shows the convergence between the LLR shapes, despite the aforementioned gap between the estimated values of b.

IV. Simulation Result

For our simulation results, we propose to use a regular (3,6) LDPC code of length $n = 20000$ over an additive SαS noise channel of parameter $\alpha = 1.4$ (high impulsiveness) and $\alpha = 1.8$ (low impulsiveness). For each channel state, a learning sequence of 20000 samples is used for the supervised optimization. In case of an impulsive environment with $\alpha < 2$, the second-order moment of a stable variable is infinite [17, Theorem 3], making the conventional noise power measurement not well-defined. Accordingly, we present our simulation results as a function of γ, which is used as a measurement of the strength of the α-stable noise.

Fig. 7 and Fig. 8 present the obtained BER in low and high impulsiveness respectively, as a function of

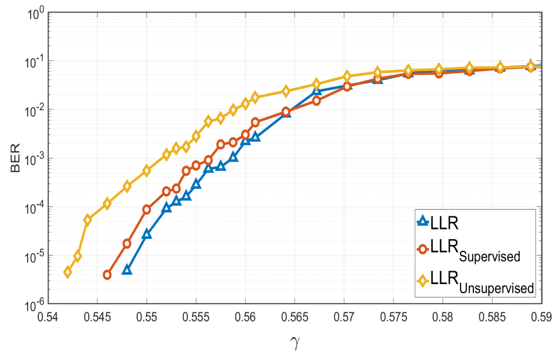

Fig. 7. Evolution comparison of the BER as a function of the dispersion γ of a SαS noise in poorly impulsive environment with $\alpha = 1.8$, in the supervised LLR approximation, the unsupervised LLR approximation and with LLR obtained by numerical integration.

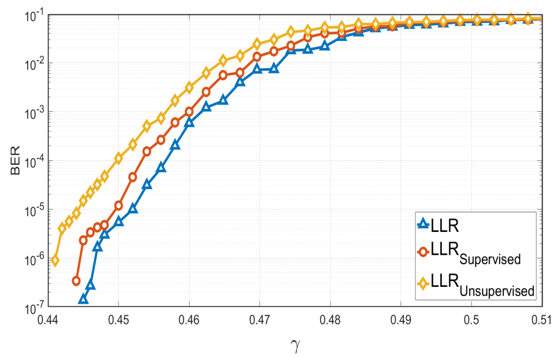

Fig. 8. Evolution comparison of the BER as a function of the dispersion γ of a SαS noise in highly impulsive environment with $\alpha = 1.4$, in the supervised LLR approximation, the unsupervised LLR approximation and with LLR obtained by numerical integration.

the dispersion γ of the alpha-stable noise. For each value of γ, we compare the BER obtained via LLR approximation, either in a supervised or unsupervised manner to the BER obtained with the true LLR. First, note that the unsupervised method performs close to the supervised case. One major advantage of the blind optimization, besides the absence of a learning sequence, relies on the direct use of the noise impacting the codeword's transmission, reducing thus the risk of a bad noise estimation, especially if the noise varies quickly from one packet to another. Moreover, the gap between the estimated LLR and exact LLR is close in both environments, proving the strength of our proposed method, and the ability to adapt to the change of impulsiveness.

V. Conclusion

In this paper, we proposed an unsupervised LLR approximation used as the input of an LDPC decoder over an additive symmetric α-stable noise channel. Our new unsupervised approximated LLR demapper features an easy implementation, thanks to the simplicity of our proposed noise extraction used in the optimization step. Moreover, the performance obtained with our solution is very close to the one obtained using a supervised LLR approximation or the one obtained with the true LLR, proving the strength of our blind approximation. Indeed, a very small gap to the true LLR is achieved without decreasing the throughput and without requiring numerical integration.

Acknowledgments

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