On the multiway relay channel with direct links

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***Abstract*—This paper studies the extension of the multiway relay channel model with restricted encoders (introduced by Gündüz et al.) by adding unit-gain intra-cluster links. In this model, multiple clusters of users communicate with the help of one relay and the users within a cluster wish to exchange messages among themselves. We obtain achievable rates and gaps to the cut-set bound as a function of the number of users and the cluster-to-relay gain g.**

I. Introduction

Many different protocols have been proposed to communicate over the Gaussian relay channel, such as Decode-and-Forward (DF), Amplify-and-Forward (AF), Compress-and-Forward (CF) [1, Ch. 16]. A natural extension of this three-node channel is the Two-Way Relay Channel (TWRC), where two users wish to exchange messages with the help of a relay. The TWRC without [1] and with [2], [3], [4] direct links between the two users has been extensively studied and DF, AF and CF have been adapted to this channel. The Compute-and-Forward (CoF) protocol by Nazer et al. [5], which at the relay decodes the sum of the messages instead of the individual messages, has also been proposed for this channel [6].

An extension of the TWRC called the multiway relay channel has been proposed by Gündüz et al. in [7]: they consider multiple clusters of users that wish to exchange messages locally within each cluster, with the help of a single relay. Achievable rates for DF, CF, AF and CoF are given for the so-called restricted model, in which the nodes' channel inputs depend only on their own messages, not on past symbols.

The main difference between their multiway relay channel and the model in this paper is that we consider the presence of direct links between users of the same cluster. Our main contribution is the characterization of achievable rates for DF, CF, AF protocols and the cut-set bound. Gaps between this upper bound and the different protocols are also given. We also consider CoF when there are only 2 users per cluster.

II. System model

This paper considers a Gaussian multiway relay channel (mRC) in which N users, grouped into L clusters of $K \geq 2$ users each ($N = KL$), exchange messages with the help of one relay. The K users in each cluster want to recover the messages of all other users within their cluster. We consider the case when users in a cluster receive each other's transmitted signals, which may model a sensor network. We also assume that the relay has a better observation of the transmitted messages than the users and model this assumption through a non-unitary gain $g > 1$ on links between the relay and the users (this can

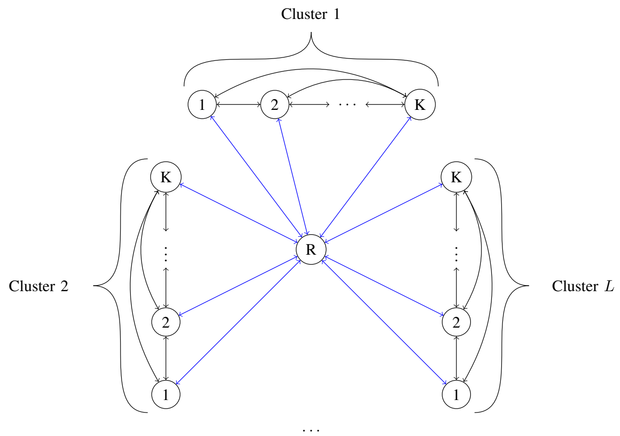

Fig. 1. Model setup: L clusters of K users each, fully connected within a cluster, communicating over one relay; user-relay links (blue) have gain g

be justified by better antennas and/or higher power and less noise, i.e. more powerful hardware at the relay). All nodes are full-duplex: they can receive and send at the same time. This situation is depicted in Fig. 1.

Our notation closely follows the one in [7]. User k of cluster l is denoted by $T_{l,k}$ and the relay is denoted by R. The message of user $T_{l,k}$ is denoted by $M_{l,k} \in \mathcal{M}_{l,k}$. User $T_{l,k}$ wants to recover the messages $(M_{l,1}, \dots, M_{l,K})$. The full-duplex Gaussian multiway relay channel is modeled as

$$Y_{j,i} = \sum_{k \neq i}^{K} X_{j,k} + gX_R + Z_{j,i} \tag{1}$$

$$Y_R = \sum_{l=1}^{L} \sum_{k=1}^{K} gX_{l,k} + Z_R \tag{2}$$

where Z_R and $Z_{j,i}$ are zero-mean, unit-variance Gaussian noises that are independent of each other and the channel inputs. The difference with the model in [7] is the presence of the intra-cluster signal in (1).

A $(2^{nR_{1,1}}, \dots, 2^{nR_{L,K}}, n)$-code for the multiway relay channel with restricted encoders consists of

- N messages sets $\mathcal{M}_{l,k} = \{1, \dots, 2^{nR_{l,k}}\}$
- N user node encoding functions $x_{l,k}^n = f_{l,k}(M_{l,k})$
- A relay encoding function $x_R = f_R(Y_R)$
- N decoding functions $g_{l,k} : \mathcal{Y}_{l,j} \times \mathcal{M}_{l,k} \rightarrow (\mathcal{M}_{l,1}, \dots, \mathcal{M}_{l,K})$

All messages $M_{l,k}$ are assumed to be chosen independently and uniformly over $\mathcal{M}_{l,k}$.

We focus on a symmetric network where each user has the same power constraint P and the relay has power constraint P_R, for which we characterize the achievable equal rate points (the *exchange rate* defined in [7]): $R_{l,k} = R, \forall(l,k)$. Total exchange rate R_t is achievable for the mRC with L clusters of K users each if $(\frac{R_t}{KL}, \dots, \frac{R_t}{KL})$ is an achievable rate tuple.

As in [7]), we suppose that each cluster transmits only over a 1/L time slot. We use the notation $C(x) = \frac{1}{2}\log_2(1+x)$.

III. Upper bound and achievable rates

Since each cluster communicates only over a 1/L time slot, we can perform all computations for one cluster and then divide the obtained results by the number of clusters. We can also increase the transmitted power of each user up to $P' = LP$ and still satisfy the average power constraint. For notational simplicity, we drop the cluster index, yielding

$$Y_i = \sum_{k\neq i}^{K} X_i + gX_R + Z_i$$

$$Y_R = g\sum_{k=1}^{K} X_k + Z_R$$

In the following, two tools are used extensively. First, we bound the exchange rates of a multiple-access channel (MAC). If we consider the MAC $Y_R = \sum_{k=1}^{K} X_k + Z$, where each X_k is of power P and Z is zero-mean Gaussian noise of variance N, the destination can decode all messages if $kR \le C(kP/N)$, $\forall k \in [1, K]$. The most restricting inequality is for $k = K$: if it is satisfied, then all other inequalities are also satisfied. Thus, the exchange rate of a MAC with K users of power P and noise power N is given by

$$R \le \frac{1}{K} C\left(KP/N\right) \tag{3}$$

The second tool is the fact that $\mathrm{E}[Var(Y|X)]$ lower-bounds the linear MMSE estimate of Y given X.

A. Cut-set bound

Proposition 1: For a symmetric Gaussian multiway channel with direct links, L clusters of K users each, the cut-set bound is given by:

$$R \le \max_{\rho\in[0,1]} \frac{K}{K-1} \min\left\{f_1(\rho), f_2(\rho)\right\} \tag{4}$$

where

$$f_1(\rho) = C\left(\frac{(g^2+1)P'(K-1)\left((1-\rho^2) - (K-1)\rho^2\right)}{1-\rho^2}\right) \tag{5}$$

$$f_2(\rho) = C\left((K-1)P' + g^2P_R(1-\rho^2) + 2g\sqrt{P'P_R}(K-1)\rho\right) \tag{6}$$

Proof: Arguing that the most restricting cut is the one with all nodes but one in a set, we obtain

$$\begin{cases}(K-1)R \le I(X_1, \dots, X_{K-1}; Y_K, Y_R | X_K, X_R)\\(K-1)R \le I(X_1, \dots, X_{K-1}, X_R; Y_K | X_K)\end{cases} \tag{7}$$

Using MMSE estimates and the exchange rate constraint for a MAC with $K{-}1$ users, we obtain Proposition 1. Here ρ stands for the correlation coefficient $\rho = E[X_iX_R]/\sqrt{E[X_i^2]E[X_R^2]}$. ■

Proposition 2: For a symmetric Gaussian mRC with direct links, L clusters of K users each, the cut-set bound can be upper-bounded by:

$$R_{CSB} \le \frac{K}{2(K-1)} \log_2\left(1 + (g^2+1)(K-1)P'\right) \tag{8}$$

Proof sketch: The proof proceeds as follows. Show that $f_1(\rho)$ is a decreasing function and $f_2(\rho)$ is either a concave function if $\frac{K-1}{g}\sqrt{\frac{P'}{P_R}} \le 1$ or a strictly increasing function. If $P_R \le (K-1)P'$, then $f_1(0) \ge f_2(0)$ and $f_1(1) \le f_2(1)$, thus there exists one intersection point and (8) holds. If $P_R \ge (K-1)P'$, either $f_1(\rho) \le f_2(\rho)$ or there is at least one intersection point, thus (8) holds. ■

In the following subsections we characterize exchange rates for various relaying schemes, such as Amplify-and-forward (AF), Decode-and-forward (DF), Compress-and-Forward(CF).

B. Compress-and-forward

This part is inspired by [8], where a lattice-based CF scheme has been proposed for the Gaussian relay channel, in which a single node wishes to transmit his message to a destination with the help of a relay. Here we extend the proposed scheme to multiple nodes in a cluster.

Proposition 3: For a symmetric Gaussian mRC with direct links, L clusters of K users each, the following exchange rate is achievable with CF relaying using lattice codes:

$$R \le \frac{K}{K-1} C\left((K-1)P'\left(1 + \frac{g^2}{1+D}\right)\right), \tag{9}$$

with

$$D = \frac{(1+g^2)(K-1)P' + 1}{g^2 P_R}. \tag{10}$$

Proof: Codebooks for all transmitters k: $c_{tk} \in \mathcal{C}_t = \{\Lambda_{c_t} \cap \mathcal{V}_c\}$ where $\Lambda_c \subseteq \Lambda_{c_t}$ are nested lattices and Λ_c is both Rogers- and Poltyrev-good and Λ_{c_t} is Poltyrev-good. To ensure the power constraints, we choose $\sigma^2(\Lambda_c) = P'$. Each message $w_k \in \{1, \dots, 2^{nR}\}$ is associated with one codeword c_{tk}.

Codebook for the relay: $c_R \in \mathcal{C}_R = \{\Lambda_{c_R} \cap \mathcal{V}_R\}$ where $\Lambda_R \subseteq \Lambda_{c_R}$ and Λ_R is both Rogers- and Poltyrev-good and Λ_{c_R} is Poltyrev-good. To ensure the power constraints, we choose $\sigma^2(\Lambda_R) = P_R$. Each compression index $i \in \{1, \dots, 2^{nR'}\}$ is associated with one c_R.

Quantization codebook: $c_q \in \mathcal{C}_q = \{\Lambda_q \cap \mathcal{V}\}$ where Λ_q is Rogers-good and Λ is Poltyrev-good. We choose $\sigma^2(\Lambda_q) = D$ and $\sigma^2(\Lambda) = 1 + D + \frac{g^2(K-1)P'}{1+(K-1)P'}$. The compression rate is thus $R_q = \frac{1}{2}\log_2\left(\frac{\sigma^2(\Lambda)}{\sigma^2(\Lambda_q)}\right)$.

We use block Markov encoding. In block b, user k sends

$$X_k(w_{b,k}) = [c_{tk}(w_{b,k}) + U_{tk}(b)] \mod \Lambda_c \tag{11}$$

where $U_{tk}(b)$ is a dither uniformly distributed over $\mathcal{V}_c$.

At the relay, the received signal is compressed to $I(b–1) = \left[Q_q\left(g\sum_{k=1}^{K} X_k(w_{b-1,k}) + Z_R(b-1) + U_q(b-1)\right)\right] \mod \Lambda$, where U_q is a quantization dither uniformly distributed over $\mathcal{V}_q$.

$$I(b-1) = \left[g\sum_{k=1}^{K} X_k(w_{b-1,k}) + Z_R(b-1) \right.$$
$$\left. + U_q(b-1) - E_q(b-1)\right] \mod \Lambda \quad (12)$$

where E_q is the quantization error. The relay chooses the codeword $c_R(i(b-1))$ associated with the index $i(b-1)$ of $I(b-1)$ and sends

$$X_R(b-1) = [c_R(i(b-1)) + U_R(b-1)] \mod \Lambda. \quad (13)$$

User i receives $Y_i(b) = \sum_{k\neq i}^{K} X_k(w_{b,k}) + gX_R(b-1) + Z_i(b)$.

First, it decodes X_R, which can be done if

$$R_q \leq \frac{1}{2}\log_2\left(1 + \frac{g^2 P_R}{1+(K-1)P'}\right). \quad (14)$$

Then it can remove it, forming $\tilde{Y}_i(b) = \sum_{k\neq i}^{K} X_k(w_{b,k}) + Z_i(b)$, and uses $\tilde{Y}_i(b-1)$ as side information to reconstruct $\hat{Y}_R(b-1)$ as:

$$\hat{Y}_R(b-1) = g\sum_{k\neq i}^{K} X_k(w_{b-1,k}) + Z_R(b-1) - E_q(b-1). \quad (15)$$

Then user i decodes the other messages by combining $\hat{Y}_R$ and $\tilde{Y}_i(b-1)$ as

$$\hat{Y}_R(b-1)\frac{g}{1+D} + \tilde{Y}_i(b-1) = \sum_{k\neq i}^{K} X_k(w_{b-1,k})\left(1+\frac{g^2}{1+D}\right)$$
$$+ Z_i(b-1) + (Z_R(b-1) - E_q(b-1))\frac{g}{1+D}. \quad (16)$$

Thus, it can decode $\{X_k^l\}$, $k \in [1,\ldots,K]\setminus i$ if

$$(K-1)R \leq \frac{1}{2}\log_2\left(1 + (K-1)P'\left(1+\frac{g^2}{1+D}\right)\right). \quad (17)$$

The source rate constraint leads to $D = \frac{(1+g^2)(K-1)P'+1}{g^2 P_R}$. ■

C. Decode-and-forward

This part is inspired by [9], where a DF scheme using AWGN superposition coding and decoding has been proposed for the Gaussian relay channel, in which a node wishes to transmit his message to a destination with the help of a relay. Here we extend the proposed scheme to multiple users in a cluster.

Proposition 4: For a symmetric Gaussian mRC with direct links, L clusters of K users each, the following exchange rate is achievable with DF relaying:

$$R \leq \max_{\rho\in[0,1]} \min\{R_1(\rho), R_2(\rho)\} \quad (18)$$

where

$$R_1(\rho) = C\left(g^2(1-\rho^2)KP'\right) \quad (19)$$

$$R_2(\rho) = \frac{K}{K-1}C\left((K-1)\left(P' + g^2\frac{P_R}{K} + 2g\rho\sqrt{\frac{P'P_R}{K}}\right)\right) \quad (20)$$

Proof: The codeword of each transmitter is the superposition of two codewords,

$$X_k(i_k, j_k) = \sqrt{\frac{\rho^2 P'}{\frac{P_R}{K}}}X_{k1}(i_k) + \sqrt{1-\rho^2}X_{k2}(j_k), \quad (21)$$

where i_k, j_k range from 1 to 2^{nR}. We impose the following power constraints: X_{k1} is of power P_R/K and X_{k2} of power P'. In the first block, all K nodes transmit $X_k(w_{k,1}, w_{k,2})$. Then in block 2, all K nodes transmit $X_k(w_{k,2}, w_{k,3})$ until block $K+1$, where they transmit $X_k(w_{k,K}, w_{k,K+1})$; $w_{k,1}$ is predetermined for all users. At block b, the relay receives

$$Y_R(b) = g\sqrt{\frac{\rho^2 P'}{\frac{P_R}{K}}}\sum_{k=1}^{K} X_{k1}(w_{k,b})$$
$$+ g\sqrt{1-\rho^2}\sum_{k=1}^{K} X_{k2}(w_{k,b+1}) + Z_R(b). \quad (22)$$

During the previous block, the relay has decoded all $w_{k,b}$ for $k \in [1,\ldots,K]$, so it can remove them and decode all $w_{k,b+1}$ for $k\in[1,\ldots,K]$ if

$$KR \leq \frac{1}{2}\log_2\left(1 + g^2(1-\rho^2)KP'\right). \quad (23)$$

Then the relay sends

$$X_R(b) = \sum_{k}^{K} X_{k1}(w_{k,b}). \quad (24)$$

User i receives at block b

$$Y_i(b) = \left(\sqrt{\frac{\rho^2 P'}{\frac{P_R}{K}}} + g\right)\sum_{k\neq i}^{K} X_{k1}(w_{k,b})$$
$$+ \sqrt{1-\rho^2}\sum_{k\neq i}^{K} X_{k2}(w_{k,b+1}) + Z_i(b) \quad (25)$$

It starts decoding $X_{k2}(w_{k,b})$ from $Y_i(b-1)$ and then decodes $X_{k1}(w_{k,b})$ from $Y_i(b)$. This succeeds if

$$(K-1)R \leq \frac{1}{2}\log_2\left(1 + (1-\rho^2)(K-1)P'\right)$$
$$+ \frac{1}{2}\log_2\left(1 + \frac{\left(\sqrt{\frac{\rho^2 P'}{P_R/K}} + g\right)^2 (K-1)\frac{P_R}{K}}{(1-\rho^2)(K-1)P'+1}\right)$$
$$= C\left((K-1)\left(P' + g^2\frac{P_R}{K} + 2g\rho\sqrt{\frac{P'P_R}{K}}\right)\right).$$

■

D. Amplify-and-forward

This part is inspired by [10], where an AF scheme has been proposed for the Gaussian relay channel.

Proposition 5: For a symmetric Gaussian mRC with direct links, L clusters of K users each, the following exchange rate is achievable with AF relaying:

$$R \leq \frac{K}{2(K-1)} \log_2 \left(\frac{\alpha + \sqrt{\alpha^2 - \beta^2}}{2} \right), \tag{26}$$

with

$$\begin{aligned} \alpha &= 1 + (K-1)P' \frac{g^2(KP' + g^2 P_R) + 1}{g^2(KP' + P_R) + 1} \\ \beta &= 2(K-1)P' g^2 \frac{\sqrt{P_R(g^2 KP' + 1)}}{g^2(KP' + P_R) + 1} \end{aligned} \tag{27}$$

Proof: The relay sends

$$X_R(b) = \sqrt{\frac{P_R}{g^2 KP' + 1}} \left(g \sum_{k=1}^{K} X_k(b-1) + Z_R(b-1) \right). \tag{28}$$

User i receives

$$\begin{aligned} Y_i(b) =& g^2 \sqrt{\frac{P_R}{g^2 KP' + 1}} \sum_{k \neq i} X_k(b-1) + \sum_{k \neq i} X_k(b) \\ &+ Z_i(b) + g \sqrt{\frac{P_R}{g^2 KP' + 1}} Z_R(b-1). \end{aligned} \tag{29}$$

The total noise power is: $N_{eq} = \frac{g^2(KP'+P_R)+1}{g^2 KP'+1}$. We can divide $Y_i(b)$ by $\sqrt{N_{eq}}$ to get

$$\begin{aligned} \tilde{Y}_i(b) =& \sqrt{\frac{g^2 KP' + 1}{g^2(KP' + P_R) + 1}} \sum_{k \neq i} X_k(b) \\ &+ g^2 \sqrt{\frac{P_R}{g^2(KP' + P_R) + 1}} \sum_{k \neq i} X_k(b-1) + Z_{eq}(b) \end{aligned} \tag{30}$$

where $Z_{eq}(b)$ has unit power.

Thus, the AF protocol transforms the channel into a unit-memory intersymbol MAC. The achievable rate is then given by:

$$(K-1)R \leq \frac{1}{2} \frac{1}{2\pi} \int_0^{2\pi} log_2(1 + (K-1)P' |H(\omega)|^2) d\omega, \tag{31}$$

where $H(\omega)$ is the Fourier transform of $H_k = \left[\sqrt{\frac{g^2 KP'+1}{g^2(KP'+P_R)+1}} \quad g^2 \sqrt{\frac{P_R}{g^2(KP'+P_R)+1}} \right]$. Thus,

$$\begin{aligned} &\frac{1}{2\pi} \int_0^{2\pi} log_2(1 + (K-1)P' |H(\omega)|^2) d\omega \\ &= \log_2 \left(\frac{\alpha + \sqrt{\alpha^2 - \beta^2}}{2} \right) \end{aligned} \tag{32}$$

with

$$\begin{aligned} \alpha &= 1 + (K-1)P' \frac{g^2(KP' + g^2 P_R) + 1}{g^2(KP' + P_R) + 1} \\ \beta &= 2(K-1)P' g^2 \frac{\sqrt{P_R(g^2 KP' + 1)}}{g^2(KP' + P_R) + 1} \end{aligned} \tag{33}$$

obtained from

$$\int_0^{2\pi} log_2(x + y\cos(z)) dz = 2\pi \log_2 \left(\frac{x + \sqrt{x^2 - y^2}}{2} \right) \tag{34}$$

found in [11, 4.224.9]. ■

E. Compute-and-Forward

Proposition 6: For a symmetric Gaussian mRC with direct links, L clusters of $K = 2$ users each, the following exchange rate is achievable with CoF relaying:

$$R \leq \min \left\{ \log_2^+ \left(\frac{1}{2} + g^2 P' \right), \log_2(1 + P' + g^2 P_R) \right\}. \tag{35}$$

Proof: The following nested lattices are used: $\Lambda \subseteq \Lambda_s \subseteq \Lambda_c$, $\Lambda_R \subseteq \Lambda_s R \subseteq \Lambda_c R$, with $\sigma^2(\Lambda) = P'$ and $\sigma^2(\Lambda_R) = P_R$. Transmitters sends: $X_i(b) = [t_i(b) + U_i(b)] \mod \Lambda$, where $U_i(b)$ is a dither uniformly distributed over $\mathcal{V}$.

The relay receives $Y_R = g(X_1 + X_2) + Z_R$ and computes $T = [t_1 + t_2] \mod \Lambda$ if

$$R \leq \frac{1}{2} \log_2^+ \left(\frac{1}{2} + g^2 P' \right), \tag{36}$$

where $\log_2^+(x)$ means $\log_2^+(x) = \max(\log_2(x), 0)$.

Then, using the list decoder proposed in [6], the destinations can decode each others message as long as

$$R \leq \frac{1}{2} \log_2(1 + P' + g^2 P_R).$$

■

IV. Gap to cut-set bound

Proposition 7: For a symmetric Gaussian mRC with direct links, L clusters of K users each, the CF protocol achieves rates within $\frac{K}{2(K-1)} \log_2(1+g^2)$ bits of the exchange capacity.

Proof:

$$\begin{aligned} R_{CF} &= \frac{K}{2(K-1)} \log_2 \left(1 + (g^2+1)(K-1)P'\right) \\ &+ \frac{K}{2(K-1)} \log_2 \left(\frac{1 + (K-1)P' + g^2 P_R}{1 + (1+g^2)(K-1)P' + g^2 P_R} \right) \\ &\geq R_{CSB} - \frac{K}{2(K-1)} \log_2 \left(\tfrac{1+(1+g^2)(K-1)P'+g^2 P_R}{1+(K-1)P'+g^2 P_R} \right) \\ &\geq R_{CSB} - \frac{K}{2(K-1)} \log_2(1+g^2), \end{aligned} \tag{37}$$

where R_{CSB} is the cut-set rate (4). The last inequality is obtained by performing the analysis of $\frac{1+(1+g^2)(K-1)P'+g^2 P_R}{1+(K-1)P'+g^2 P_R}$ as a function of P and P_R. ■

Similar results can be obtained for the DF and AF protocols.

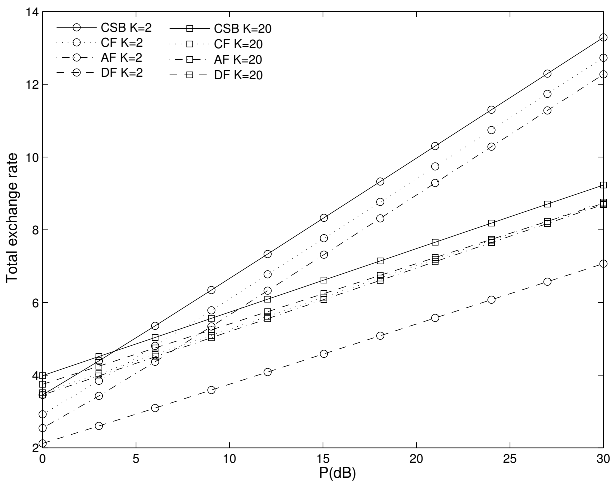

Fig. 2. Total exchange rate vs. P, $P_R = KP$, $g = 3$, $L = 1$

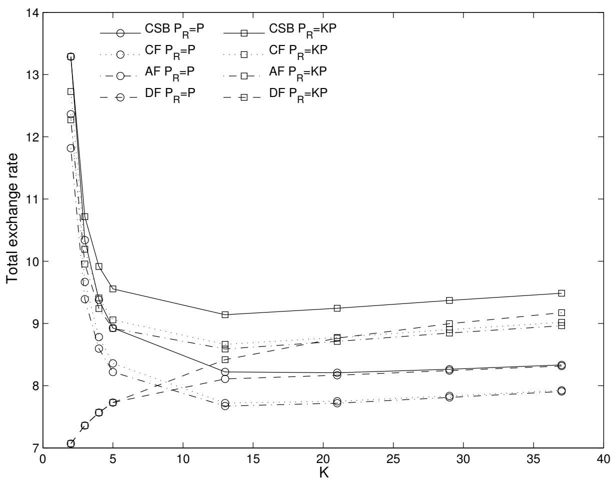

Fig. 3. Total exchange rate vs. K, $P = 30dB$, $g = 3$, $L = 1$

V. Results

In Fig. 2, we plot the cut-set bound, the achievable symmetric rate for the mRC with $L = 1$ cluster of $K = 2$ and $K = 20$ users as a function of P. We can notice the finite gap between the cut-set bound and the CF protocol at all power levels, we also notice that AF follows CF with a constant gap. For a small number of users, CF dominates DF.

In Fig. 3, we plot the cut-set bound, the achievable symmetric rate for the mRC with $L = 1$ cluster and $P = 30dB$ as a function of K. We observe that DF achieves the cut-set bound when the relay power doesn't scale with the number of users.

In Fig. 4, we plot the cut-set bound, the achievable symmetric rate for the mRC with $L = 8$ clusters of 2 users as a function of P. We can note that for the chosen g, CoF gives the best performances among the proposed schemes. We can also see that the gap between the cut-set bound and CoF tends to zero for high power P.

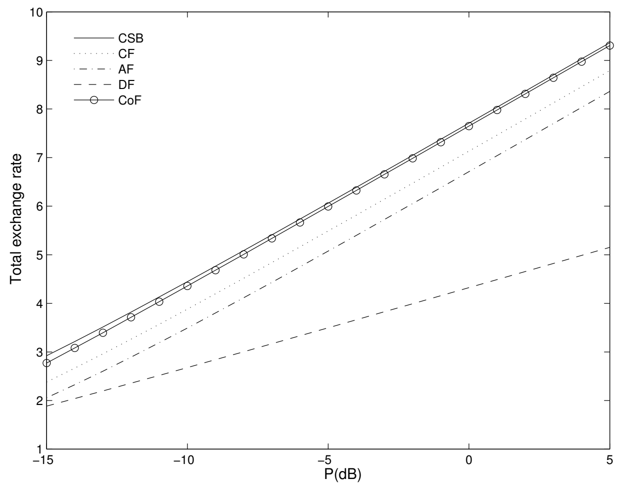

Fig. 4. Total exchange rate vs. P, $P_R = 2LP$, $g = 5$, $L = 8$

VI. Discussion

In this paper we considered an extension of the multiway relay channel proposed by Gündüz et al. in [7]. In our setup, multiple clusters of users with direct intra-cluster links communicate with the help of a single relay. Each user wishes to recover all messages within its cluster. Using results proposed for the Gaussian relay channel based on lattices [5], [6] [8] or standard AWGN coding/decoding [9] [10], we extended standard schemes such as CF, DF, AF for this setup. For very large user-relay gain g, i.e. when the model becomes that of [7] up to scaling, the behaviors in Fig. 2, Fig. 3, Fig. 4 become the same as the ones obtained by Gündüz et al. in [7] (by scaling the node and relay transmit powers by a factor g^2).

References

[1] A. El Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge University Press, 2011.

[2] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," *ISIT*, 2006.

[3] P. Zhong and M. Vu, "Decode-forward and compute-forward coding schemes for the two-way relay channel," *ITW*, 2011.

[4] ——, "Partial decode-forward coding schemes for the Gaussian two-way relay channel," *ICC*, 2012.

[5] B. Nazer and M. Gastpar, "Compute-and-forward: harnessing interference through structured codes," *IEEE Transactions on Information Theory*, vol. 57, pp. 6463–6486, 2011.

[6] Y. Song and N. Devroye, "Lattice codes for the Gaussian relay channel: Decode-and-forward and compress-and-forward," *IEEE Transactions on Information Theory*, vol. 59, pp. 4927 – 4948, 2013.

[7] D. Gündüz, A. Yener, A. Goldsmith, and H. V. Poor, "The multiway relay channel," *IEEE Transactions on Information Theory*, vol. 59, pp. 51–63, 2013.

[8] Y. Song and N. Devroye, "A lattice compress-and-forward scheme," *ITW*, 2011.

[9] A. Khina, O. Ordentlich, U. Erez, Y. Kochman, and G. Wornell, "Decode-and-forward for the Gaussian relay channel via standard AWGN coding and decoding," *ITW*, 2012.

[10] W. Chang, S.-Y. Chung, and Y. H. Lee, "Gaussian relay channel capacity to within a fixed number of bits," *Computing Research Repository - CORR*, 2010.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 7th ed. Elsevier/Academic Press, Amsterdam, 2007.