Optimal power allocation policies in multi-hop cognitive radio networks

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***Abstract*—This paper investigates a power allocation problem in an optimal and closed-form manner for a relay-aided cognitive network composed of one primary and secondary user/destination pair (e.g., a cellular link coexisting with a device-to-device link in a device-to-device enabled cellular communication setup) and one secondary full-duplex relay. By exploiting the monotonic properties of the objective function and the geometry of the feasible set, we provide the optimal and closed-form power allocation policy assuming the relay performs either Decode-and-Forward (DF) or Compress-and-Forward (CF). We then conjecture that if the secondary direct link is missing, then DF always outperforms CF, irrespective from the system parameters. This conjecture is sustained by numerous numerical simulations.**

***Index Terms*—cognitive radio, full-duplex relay, device-to-device enabled cellular networks**

I. Introduction

Over the wireless medium, since all receivers within range can overhear signals sent by some transmitters and due to the expected large increase of the number of communicating devices within the next generation of communication systems, exploiting both cooperative communications and device-to-device enabled cellular communications emerge among the most promising ways to better exploit the network capacity [1], [2]. Moreover, from an energy-efficiency perspective, wireless power transfer among interconnected devices is also envisioned for future communications [3], [4].

The most basic model of a cooperative communication is the relay channel, where a node, called relay, is willing to help the communication between a given source and its associated destination [5]. Three main relaying schemes have been proposed in the literature: Decode-and-Forward (DF), Compress-and-Forward (CF) and Amplify-and-Forward (AF). Unfortunately, none of these relaying schemes performs best in general over the relay channel, nor over various extensions, such as the two-way relay channel [6], the diamond relay channel [7], or the multiway relay channel [8].

Given the above considerations, the aim of this paper is to study an optimal power splitting policy among an opportunistic device and its helping relay operating in a full-duplex manner, and to evaluate the impact of an ideal power transfer protocol between the two devices by considering an overall power budget [9]. Furthermore, a cognitive radio context is assumed in which the opportunistic user (e.g., a device-to-device link) is allowed to communicate over the primary spectrum provided the primary link (e.g., a cellular user link to the base station) is not disturbed. We study a minimum Quality of Service (QoS) constraint to protect the primary user [10], different than the more common maximum interference constraints [11], allowing the secondary user to transmit as long as the primary user achieves its desired target Shannon rate. Throughout this paper, we focus on CF and DF; AF is not considered here due to its poor performance in multi-user interference settings (the relay amplifies not only the useful signal but also the noise plus interference).

Existing works on power allocation problems in relay-aided cognitive radio networks include [12]–[16]. Among them, [12], [13] investigate the minimization of an outage probability metric, while we focus on rate-driven communications. In [14]–[16], the helping relay nodes use simple AF. Finally, all these works consider peak interference constraints to protect the primary link, while we focus on a minimum QoS constraint. Other works including [17]–[20] consider relay-aided cognitive radio networks such that no interference from the primary network impacts the secondary network, which we do not ignore here.

The closest work to the present paper is our previous study [10], in which we investigate the power allocation problem assuming that the interfering links between the primary and secondary users are negligible. Here, we no longer make this simplifying assumption. Instead, we assume that no direct link exists between the secondary user and its destination, so that the secondary transmission needs to go through the relay. Such a situation can arise for instance when the secondary user is too far apart from its destination (rending the channel gain negligible) or when no line of sight exists between these two nodes. Aside from the communication setup, the main difference with [10] lies in the available power profile. Here, we investigate the possibility of having an overall power constraint among the opportunistic user and the relay as opposed to having individual power constraints. Such a constraint [9] enables us to asses the theoretical limits of the future wireless

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power transfer technologies.

Our main contributions can be summarized as follows. First, we derive the optimization problem under both DF and CF relaying when an overall power constraint applies on the secondary network. We then provide the optimal power allocation policy in a closed-form under both relaying schemes. Finally, we conjecture that DF always outperforms CF for all channel setups, due to the lack of side information coming from a direct link leading to a coding scheme solely based on scaling.

II. System model and optimization problem

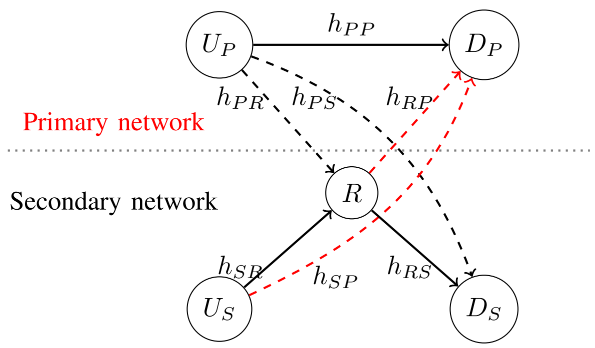

Fig. 1. Cognitive relay-aided network.

The cognitive network under study is illustrated in Fig. 1. The primary network is composed by a primary user U_P and its associated destination D_P modeling for instance a cellular link between a cellular user and the base station. The secondary network is composed by a user U_S, its destination D_S and a relay R, modeling a relay-aided device-to-device communication link. The relay node operates in a full-duplex mode such that it can simultaneously receive and transmit information, and it can perfectly cancel out any self-interference.

The received signals at the relay and at the two destinations write as

$$Y_R = h_{PR}X_P + h_{SR}X_S + Z_R, \tag{1}$$
$$Y_i = h_{Ri}X_R + h_{ii}X_i + h_{ji}X_j + Z_i, \tag{2}$$

where $i \in \{P, S\}$, $j = \{P, S\} \setminus i$, X_P, X_S and X_R denote the transmitted signals of the primary user, secondary user and the relay, respectively; Z_R and Z_i are the additive white Gaussian noise (AWGN) at the relay and at destination D_i of variance N_R and N_i respectively; h_{ij}, $i, j \in \{P, S, R\}$ is the channel gain between transmitter i and destination j. We further denote by P_P, P_S, and P_R the average powers of the input signals X_P, X_S, and X_R, respectively; and assume that the secondary network is such that $P_S + P_R \leq \overline{P}$. Throughout the paper, we assume that the secondary user and destination are too far apart from each other, so that no direct link between these two nodes exists, i.e. $h_{SS} = 0$.

We moreover consider a typical Block Markov coding such that, during each block k, the nodes receive and can process the messages sent during the previous block $k-1$.

Primary QoS constraint: Let $\overline{R_P}$ denote the achievable rate of the primary user in the absence of the secondary network equal to $\overline{R_P} = \frac{1}{2}\log\left(1 + \frac{h_{PP}^2 P_P}{N_P}\right)$, and let R_i, $i \in \{S, P\}$ denote the achievable rate of the secondary and primary user respectively in the presence of the secondary network.

In this paper, we aim at maximizing the achievable secondary rate R_S under a QoS constraint protecting the primary transmission [10] given as $R_P \geq (1-\tau)\overline{R_P}$, meaning that the primary user can tolerate at most a proportional ($\tau \in [0, 1]$) decrease in its achievable rate.

Problem formulation: To sum up, the optimization problem under study writes as

$$\begin{aligned} \max_{P_R, P_S} \quad & R_S(P_S, P_R) \\ \text{s.t.} \quad & R_P \geq (1-\tau)\overline{R_P}, \\ & P_R + P_S \leq \overline{P}, P_S \geq 0, P_R \geq 0, \end{aligned} \tag{3}$$

where the achievable rate of the secondary user, $R_S(P_S, P_R)$, will depend on the specific relaying scheme (DF or CF).

Notation: We use the well-known capacity function $C(x) = \frac{1}{2}\log_2(1 + x)$. Also, to simplify the mathematical expressions and derivations, we introduce the following notations, which are constants depending on the system parameters: $g_{ij} = h_{ij}^2,\ i, j \in \{P, R, S\}$;

$$\mathcal{A} = \frac{g_{PP}P_P}{\left(1 + \frac{g_{PP}P_P}{N_P}\right)^{(1-\tau)} - 1} - N_P.$$

Throughout the paper, we consider the message sent by the primary user as additional noise at both the relay and the secondary destination. Thus, one can consider an equivalent Gaussian noise at the relay, resp. the secondary destination, of variance $\widetilde{N}_R = g_{PR}P_P + N_R$, resp. $\widetilde{N}_S = g_{PS}P_P + N_S$. Similarly, the message from the secondary user and the relay are treated as additional noise at the primary destination and thus one can consider an equivalent noise term of variance $\widetilde{N}_P = h_{RP}^2 P_R + h_{SP}^2 P_S + N_P$ at node D_P.

III. DF relaying

We start by analyzing Decode-and-Forward (DF), where the relay first decodes the message sent by the opportunistic user and then re-encodes it. We provide the achievable rate region and the optimal power allocation policy in closed form.

Proposition 1 *Assuming DF at the relay and that all non-intended messages are treated as additional noise, the following rate region is achievable over the cognitive relay-aided network*

$$R_P \leq C\left(\frac{g_{PP}P_P}{\widetilde{N}_P}\right), R_S \leq C\left(\min\left\{\frac{g_{SR}P_S}{\widetilde{N}_R}, \frac{g_{RS}P_R}{\widetilde{N}_S}\right\}\right).$$

Proof: The primary achievable rate is obtained by considering the message sent by the relay and the secondary user as additional noise at the primary destination. The secondary achievable rate is obtained by considering perfect decoding at both the relay and the secondary destination when treating the message from the primary user as additional noise. ■

Having derived the achievable rates over the cognitive relay-aided network, we can specify the optimization problem (3) as follows

$$\begin{aligned} \max_{P_R, P_S} \quad & \min\left\{\frac{g_{SR}P_S}{\widetilde{N}_R}, \frac{g_{RS}P_R}{\widetilde{N}_S}\right\} \\ \text{s.t.} \quad & P_R + P_S \leq \overline{P}, 0 \leq P_R, P_S, \\ & g_{RP}P_R + g_{SP}P_S \leq \mathcal{A} \end{aligned} \tag{4}$$

In order to solve the above optimization problem, let us first focus on studying the linear constraints, which can be rewritten respectively as:

$P_R = \delta\overline{P}$ and $P_S = (1-\delta)\overline{P}$, with $\delta \in [0,1]$;
$P_R = \gamma\frac{\mathcal{A}}{g_{RP}}$ and $P_S = (1-\gamma)\frac{\mathcal{A}}{g_{SP}}$, with $\gamma \in [0,1]$.

Depending on the above constraints, four cases can arise that are depicted in Fig. 2: either the two constraints intersect or one of the constraints always dominates the other. The conditions on the system parameters that ensure the intersection of the constraints are either ($\mathcal{A} < g_{RP}\overline{P}$ and $\mathcal{A} > g_{SP}\overline{P}$), which we will refer to as assumption **[H1]** henceforth, or ($\mathcal{A} > g_{RP}\overline{P}$ and $\mathcal{A} < g_{SP}\overline{P}$), which we will refer to as **[H2]**. Under both **[H1]** and **[H2]**, the intersection point is given as

$$P_S = (1-\overline{\delta})\overline{P} = (1-\overline{\gamma})\frac{\mathcal{A}}{g_{SP}}, P_R = \overline{\delta P} = \overline{\gamma}\frac{\mathcal{A}}{g_{RP}} \text{ with}$$

$$\overline{\delta} = \frac{\mathcal{A} - g_{SP}\overline{P}}{(g_{RP}-g_{SP})\overline{P}} \text{ and } \overline{\gamma} = \frac{g_{RP}(g_{SP}\overline{P}-\mathcal{A})}{\mathcal{A}(g_{SP}-g_{RP})}, \tag{5}$$

and is such that $0 \leq \overline{\delta} \leq 1$ and $0 \leq \overline{\gamma} \leq 1$. If the following conditions are met ($\mathcal{A} \leq g_{RP}\overline{P}$ and $\mathcal{A} \leq g_{SP}\overline{P}$), i.e., **[H3]**, the primary QoS constraint is the most restricting one, whereas if either ($\mathcal{A} > g_{RP}\overline{P}$ and $\mathcal{A} \geq g_{SP}\overline{P}$) or ($\mathcal{A} = g_{RP}\overline{P}$ and $\mathcal{A} > g_{SP}\overline{P}$), i.e., **[H4]**, the total power constraint is the most restricting one.

Moreover, we can prove that the objective function is increasing in P_R for a fixed P_S and is also increasing in P_S for a fixed P_R. This implies that the optimal power allocation policy lies on the Pareto-boundary of the feasible set depicted in green in Fig. 2. We can first restrict the search for the optimal power allocation policy on one of the two linear constraints and then we take into account the overall feasible set to derive the global solution. In order to do so, we replace P_R and P_S with their parametric description based on δ and γ and define two functions $f_{DF,QoS}(\gamma)$ and $f_{DF,pow}(\delta)$ corresponding to the objective function restricted only by the primary QoS constraint and by the overall power constraint, respectively. The two functions write as

$$f_{DF,QoS}(\gamma) = \min\left\{\frac{g_{SR}g_{RP}(1-\gamma)}{g_{SP}\widetilde{N}_R}; \frac{g_{RS}\gamma}{\widetilde{N}_S}\right\}$$
$$f_{DF,pow}(\delta) = \min\left\{\frac{g_{SR}(1-\delta)}{\widetilde{N}_R}; \frac{g_{RS}\delta}{\widetilde{N}_S}\right\}$$

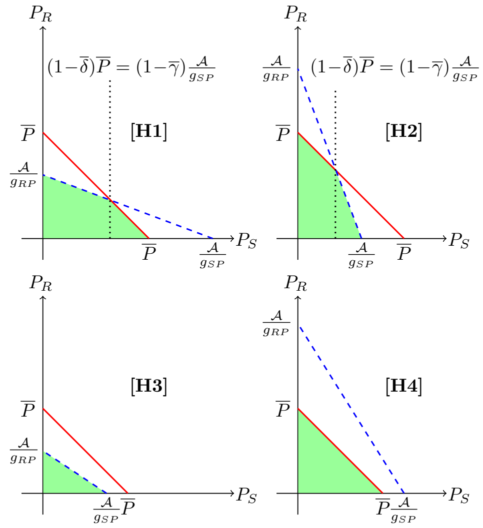

Fig. 2. Four different cases can arise when considering the primary QoS constraint (dashed line) and the total power constraint (solid line).

Theorem 1 *The optimal power allocation policy when the relay performs DF, assuming all non-intended messages as treated as additive noise, is given in closed-form as follows:*

*If **[H1]** is met, i.e. the two constraints are restrictive and intersect at a unique point: The optimal solution depends on two parameters γ^* and δ^* given as $\gamma^* = \max\{\widehat{\gamma}, \overline{\gamma}\}$ and $\delta^* = \min\{\widehat{\delta}, \overline{\delta}\}$, where $\widehat{\gamma}$ and $\widehat{\delta}$ are given at the end of this theorem. Now, if $f_{DF,pow}(\delta^*) \geq f_{DF,QoS}(\gamma^*)$, then the optimal power allocation is $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$, otherwise $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$, $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*If **[H2]** is met, the two constraints are restrictive and intersect at a unique point similarly to **[H1]** and $\gamma^* = \min\{\widehat{\gamma}, \overline{\gamma}\}$ and $\delta^* = \max\{\widehat{\delta}, \overline{\delta}\}$. Now, if $f_{DF,pow}(\delta^*) \geq f_{DF,QoS}(\gamma^*)$, then $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$, otherwise $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$ $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*If **[H3]** is met, only the QoS constraint is impacting the power allocation policy with $\gamma^* = \widehat{\gamma}$ and $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$, $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*Otherwise, if **[H4]** is met, only the overall power constraint is impacting the power allocation policy and $\delta^* = \widehat{\delta}$ and $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$.*

The parameters $\widehat{\delta}$ and $\widehat{\gamma}$ above are defined by

$$\widehat{\delta} = \frac{g_{SR}\widetilde{N}_S}{g_{SR}\widetilde{N}_S + g_{RS}\widetilde{N}_R} \textit{ and } \widehat{\gamma} = \frac{g_{SR}g_{RP}\widetilde{N}_S}{g_{SR}g_{RP}\widetilde{N}_S + g_{RS}g_{SP}\widetilde{N}_R}.$$

Proof: First note that one can treat the four cases separately and that both **[H1]** and **[H2]** combine the results obtained under **[H3]** and **[H4]**. We will thus start by considering the latter two cases.

*Under assumption **[H3]***, the optimization problem (4) reduces to $\max_{0\leq\gamma\leq 1} f_{DF,QoS}(\gamma)$. Note that $\frac{g_{RS}\gamma}{\widetilde{N}_S}$ is increasing with γ from 0 to $\frac{g_{RS}}{\widetilde{N}_S}$; whereas $\frac{g_{SR}g_{RP}(1-\gamma)}{g_{SP}\widetilde{N}_R}$ is decreasing with γ from $\frac{g_{SR}g_{RP}}{g_{SP}\widetilde{N}_R}$ to 0. Thus, the optimal choice of γ is such that both functions are equal, i.e. $\gamma^* = \widehat{\gamma}$. The objective function is thus increasing over the interval $[0, \widehat{\gamma}]$ and decreasing over $]\widehat{\gamma}, 1]$.

*Under **[H4]***, the optimization problem (4) reduces to a similar problem as under **[H3]** by choosing the objective function to be $f_{DF,pow}(\delta)$ and the optimization domain as $0 \leq \delta \leq 1$, which leads to $\delta^* = \widehat{\delta}$.

*Under **[H1]** and **[H2]***, one needs to combine the previous results with the domain constraints for δ and γ, which are given as $0 \leq \delta \leq \overline{\delta}$, $\overline{\gamma} \leq \gamma \leq 1$ under **[H1]** and $\overline{\delta} \leq \delta \leq 1$ $0 \leq \gamma \leq \overline{\gamma}$ under **[H2]**. In both cases, the objective function is given as $\max\left\{\max_{\delta} f_{DF,pow}(\delta), \max_{\gamma} f_{DF,QoS}(\gamma)\right\}$. ■

Although a bit tedious, Theorem 1 completely describes the optimal solution in a closed-form manner and depends only on the system parameters.

IV. CF SCHEME

In this section, we investigate Compress-and-Forward (CF) relaying. Similarly to DF relaying, we first provide the achievable rate region and then the optimal power allocation policy.

For simplicity of presentation, the following notations will be used throughout this section: $c_1 = \overline{P}\left(g_{RS}\widetilde{N}_R - g_{SR}\widetilde{N}_S\right)$, $c_2 = \widetilde{N}_S(\widetilde{N}_R + g_{SR}\overline{P})$, $\widetilde{c}_1 = \frac{\mathcal{A}}{g_{RP}}\left(g_{RS}\widetilde{N}_R - g_{SR}\widetilde{N}_S\frac{g_{RP}}{g_{SP}}\right)$, and $\widetilde{c}_2 = \widetilde{N}_S(\widetilde{N}_R + g_{SR}\frac{\mathcal{A}}{g_{SP}})$.

Proposition 2 *Assuming CF at the relay and that all non-intended messages are treated as additional noise, the following rate region is achievable over the cognitive relay-aided network*

$$R_P \leq C\left(\frac{g_{PP}P_P}{\widetilde{N}_P}\right), R_S \leq C\left(\frac{g_{SR}g_{RS}P_SP_R}{\widetilde{N}_R(g_{RS}P_R + \widetilde{N}_S) + \widetilde{N}_Sg_{SR}P_S}\right).$$

The proof relies on lattice coding and is outlined in the Appendix.

Given the achieved rate region above, the optimization problem writes as

$$\begin{array}{ll} \max\limits_{P_R, P_S} & \dfrac{P_SP_R}{\widetilde{N}_R(g_{RS}P_R + \widetilde{N}_S) + \widetilde{N}_Sg_{SR}P_S} \\ \text{s.t.} & P_R + P_S \leq \overline{P}, 0 \leq P_R, P_S, \\ & g_{RP}P_R + g_{SP}P_S \leq \mathcal{A} \end{array} \quad (6)$$

One can show that the objective function is increasing in P_R for a fixed P_S and increasing in P_S for a fixed P_R. Moreover, the feasible set is identical to the one in Section III and we can thus consider the four cases given in Fig. 2 separately. To derive the global solution, we can again restrict the search for the optimal power allocation policy on one of the two linear constraints and then take into account the overall feasible set. The objective function restricted only by the primary QoS constraint and by the overall power constraint, obtained by replacing P_R and P_S with their parametric description based on δ and γ, are given as

$$f_{CF,QoS}(\gamma) = \frac{(1-\gamma)\gamma}{\widetilde{c}_1\gamma + \widetilde{c}_2} \text{ and } f_{CF,pow}(\delta) = \frac{(1-\delta)\delta}{c_1\delta + c_2}.$$

Theorem 2 *The optimal power allocation policy when the relay performs CF, assuming all non-intended messages as treated as additive noise, is given in closed-form as follows.*

*If **[H1]** is met, i.e. the two constraints are restrictive and intersect at a unique point: The optimal solution depends on two parameters γ^* and δ^* given as $\gamma^* = \max\{\widehat{\gamma}, \overline{\gamma}\}$ and $\delta^* = \min\{\widehat{\delta}, \overline{\delta}\}$. Now, if $f_{CF,pow}(\delta^*) \geq f_{CF,QoS}(\gamma^*)$, then the optimal power allocation policy is $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$, otherwise $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$, $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*If **[H2]** is met, the two constraints are restrictive and intersect at a unique point similarly to **[H1]** and $\gamma^* = \min\{\widehat{\gamma}, \overline{\gamma}\}$ and $\delta^* = \max\{\widehat{\delta}, \overline{\delta}\}$. Also, if $f_{CF,pow}(\delta^*) \geq f_{CF,QoS}(\gamma^*)$, then $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$, otherwise $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$, $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*If **[H3]** is met, only the QoS constraint is impacting the power allocation policy and $\gamma^* = \widehat{\gamma}$, $P_R^* = \gamma^*\frac{\mathcal{A}}{g_{RP}}$, $P_S^* = (1-\gamma^*)\frac{\mathcal{A}}{g_{SP}}$.*

*Otherwise, if **[H4]** is met, only the overall power constraint is impacting the power allocation policy and $\delta^* = \widehat{\delta}$, $P_R^* = \delta^*\overline{P}$, $P_S^* = (1-\delta^*)\overline{P}$.*

The parameters $\widehat{\delta}$ and $\widehat{\gamma}$ above are defined by

$$\widehat{\delta} = \frac{-c_2 + \sqrt{c_2(c_1 + c_2)}}{c_1} \text{ and } \widehat{\gamma} = \frac{-\widetilde{c}_2 + \sqrt{\widetilde{c}_2(\widetilde{c}_1 + \widetilde{c}_2)}}{\widetilde{c}_1}.$$

Proof: As in Section III, the four cases can be treated separately and both **[H1]** and **[H2]** combine the results obtained under **[H3]** and **[H4]**.

*Under **[H3]***, the optimization problem (6) reduces to

$$\max_{0 \leq \gamma \leq 1} f_{CF,QoS}(\gamma).$$

The derivative of the objective function is given as

$$f'_{CF,QoS}(\gamma) = \frac{-\gamma^2\widetilde{c}_1 - \gamma\widetilde{c}_2 + \widetilde{c}_2}{(\gamma\widetilde{c}_1 + \widetilde{c}_2)^2}.$$

Solving for $f'_{CF,QoS}(\gamma) = 0$ leads to a second degree equation, which discriminant is given as $\Delta = 4\widetilde{c}_2(\widetilde{c}_2 + \widetilde{c}_1) \geq 0$: it thus admits two roots given as

$$\gamma_1 = \frac{-\widetilde{c}_2 - \sqrt{\widetilde{c}_2(\widetilde{c}_1 + \widetilde{c}_2)}}{c_1} \text{ and } \gamma_2 = \widehat{\gamma} = \frac{-\widetilde{c}_2 + \sqrt{\widetilde{c}_2(\widetilde{c}_1 + \widetilde{c}_2)}}{\widetilde{c}_1}.$$

Based on the study of the derivative $f'_{CF,QoS}(\gamma)$, one can prove that the objective function is increasing over the interval $[0, \widehat{\gamma}]$ and decreasing over the interval $]\widehat{\gamma}, 1]$.

*Under **[H4]***, the optimization problem (6) reduces to a problem of the same form as under **[H3]** by choosing the objective function as $f_{CF,pow}(\delta)$ and the optimization domain as $0 \leq \delta \leq 1$, leading to $\delta^* = \widehat{\delta}$.

*Under **[H1]** and **[H2]***, one needs to combine the previous obtained results under both **[H3]** and **[H4]** as well as the domain constraints for δ and γ, which are given as $0 \leq \delta \leq \overline{\delta}, \overline{\gamma} \leq \gamma \leq 1$ under **[H1]** and $\overline{\delta} \leq \delta \leq 10 \leq \gamma \leq \overline{\gamma}$ under **[H2]**. In both cases, the objective function is given as $\max \left\{ \max_{\delta} f_{CF,pow}(\delta), \max_{\gamma} f_{CF,QoS}(\gamma) \right\}$. ■

Similarly to DF relaying, Theorem 2 provides the optimal solution in a closed-form manner and depends only on the system parameters.

V. Comparison DF vs. CF

To the best of our knowledge, this is the first work combining full-duplex relaying jointly with opportunistic access (under a minimum QoS constraint at the primary user in terms of its Shannon rate tolerated loss) and power transfer capabilities (an overall power constraint between the secondary user and its relay). Hence, a comparison with other methods cannot be performed. Instead, we focus on evaluating the performance of DF vs. CF relaying in this setting.

Although all our numerous numerical experiments suggests that DF always outperforms CF in all settings, proving this rigorously is highly non trivial as opposed to the special case in [10] (no interference links between the secondary and primary links and individual power constraints).

Conjecture 1 *When no direct link is available in the secondary network, DF always outperforms CF under the total power constraint, irrespective from the system parameters.*

In the following, let us consider a square cell of size 1×1, where we assumed that the positions of the user/destination pairs are fixed, whereas the relay's position ranges over the cell. Moreover, we consider a path-loss channel model, such that $h_{ij} = d_{ij}^{-3/2}$ where d_{ij} denotes the distance between the nodes i and j. Fig. 3, respectively Fig. 4, depicts the gap between DF and CF as well as the achievable rate under DF relaying in function of the relay position (x_R, y_R) in the square cell when $N_P = 10, N_R = N_S = 1$, $\tau = 0.6$, $P_P = 5, \overline{P} = 10$. First, a strong interference regime (in Fig. 3) and then a weak one (in Fig. 4) is considered; interference regime is given by the relative position of the different nodes. We remark that DF achieves higher rate when the relay is close to the secondary user, as in the standard Gaussian relay channel. Moreover, in the strong interference regime, the gap between CF and DF is large for the same set of relay positions. In the weak interference setup, the set of relay positions that maximizes this gap is larger and includes the optimal DF positions.

VI. Conclusion

In this paper, we investigated a multi-hop cognitive radio network, where the transmission of the opportunistic user goes through a full-duplex operating relay node. We first characterized the achievable rate regions under both DF and CF, provided that the primary user's rate is not degraded beyond an acceptable threshold. We also assumed that the secondary nodes can exchange power among themselves up to a given total power constraint. We then exploited the monotonic properties of the objective function and the geometry of the feasible set to provide the optimal power allocation policy that maximizes the opportunistic achievable rate. Our numerical results indicate that DF always outperforms CF, irrespective from the system parameters.

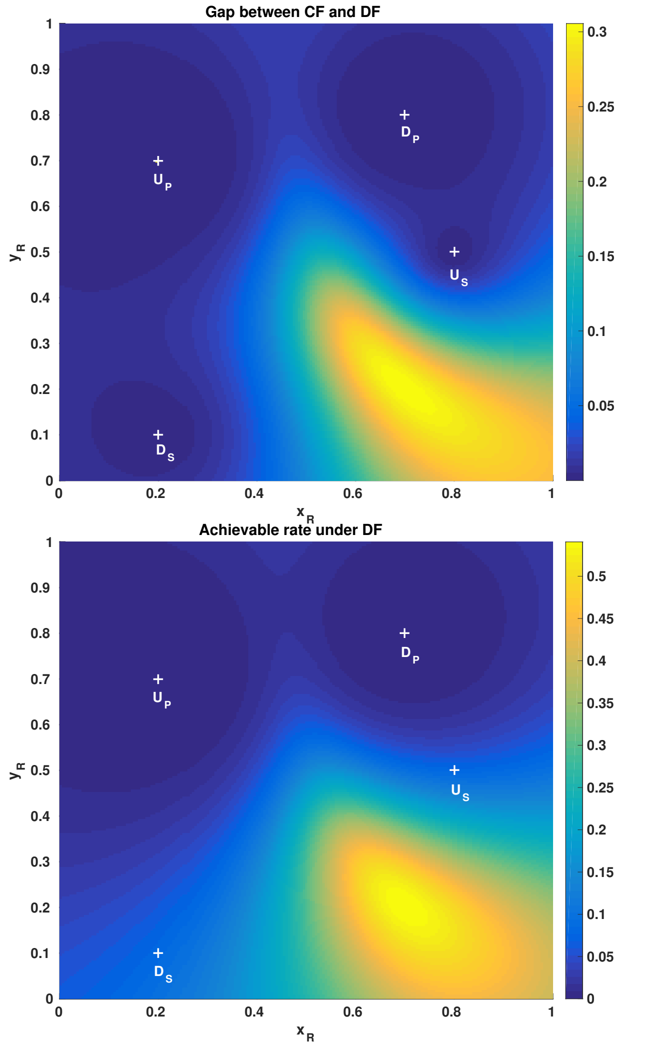

Fig. 3. Strong interference regime: gap between CF and DF (top) and achievable rate under DF (bottom). The same relay positions (x_R, y_R) give the best rate performance with DF relaying and also the largest gap between DF and CF.

Appendix: Coding Proof of Proposition 2

The proof relies on lattice coding [21].

Encoding: The proof requires the build of 3 lattice-based codebooks $\mathcal{C}_i = \{\Lambda_{c_i} \cap \mathcal{V}_i\}$, for $i \in \{S, R, Q\}$, where Q denotes the quantization. Let D denote the maximum allowed

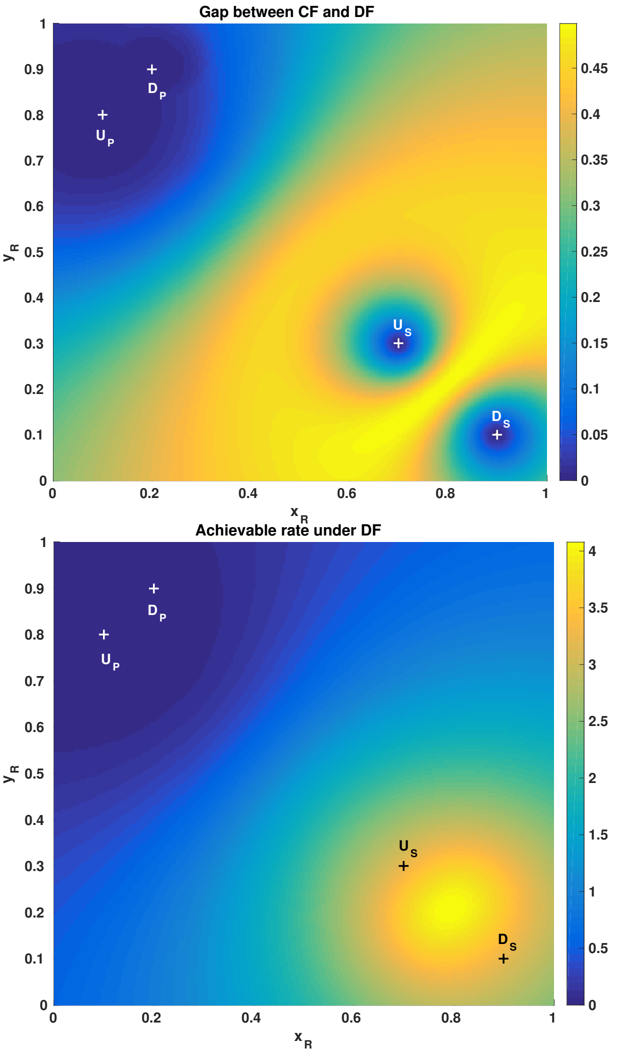

Fig. 4. Weak interference regime: gap between CF and DF (top) and achievable rate under DF (bottom). The relay positions giving the best rate performance with DF are in between the assisted transmitter and receiver. The gap between DF and CF is maximized for even more relay positions.

distortion. Throughout the proof, u_i, $i \in \{S, R, cQ\}$ denotes a dither uniformly distributed over $\mathcal{V}_i$ and known by all nodes. The codebook of the secondary user is build upon nested lattices chosen such that $|\mathcal{C}_S| = 2^{nR_S}$ and $\sigma^2(\Lambda_S) = P_S$. The quantization codebook is build upon nested lattices chosen such that $\sigma^2(\Lambda_{cQ}) = D$ and $\sigma^2(\mathit{\Lambda}_Q)$ will be specified later on in the proof. The quantization rate is thus given as $R_q = \frac{1}{2}\log_2\left(\frac{\sigma^2(\mathit{\Lambda}_Q)}{D}\right)$. During block b, the quantization index is computed as $I(b) = [\beta Y_R(b) + u_{cQ}(b) + E_{cQ}(b)] \bmod \mathit{\Lambda}_Q$, where E_{cQ} denotes the quantization error, and β is a scaling factor that will be specified later on in the proof. The relay codebook is build upon nested lattices chosen such that $\sigma^2(\Lambda_R) = P_R$. Each compression index $I \in \mathcal{C}_q$ is mapped to a single relay codeword $c_R \in \mathcal{C}_R$, in other words Λ_R is such that $|\mathcal{C}_R| = 2^{nR_q}$. *Decoding:* At the primary destination, the message from the relay and from the secondary user is treated as additional noise. The secondary destination starts by recovering the quantization index, which is possible as long as $R_q \leq C\left(\frac{g_{RS}P_R}{\tilde{N}_S}\right)$. Then, the secondary destination decodes X_S from the estimation of the received signal $\widehat{Y}_R(b) = \beta^2 Y_R(b) + \beta E_{cQ}(b)$, which requires that $\sigma^2(\mathit{\Lambda}_Q) \geq \beta^2\left(g_{SR}P_S + \tilde{N}_R\right) + D$ and $R_S \leq C\left(\frac{\beta^2 g_{SR}P_S}{\beta^2\tilde{N}_R + D}\right)$. In order to satisfy the distortion criterion, β is chosen as $\beta^2 = 1 - \frac{D}{g_{SR}P_S + \tilde{N}_R}$, which completes the proof.

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